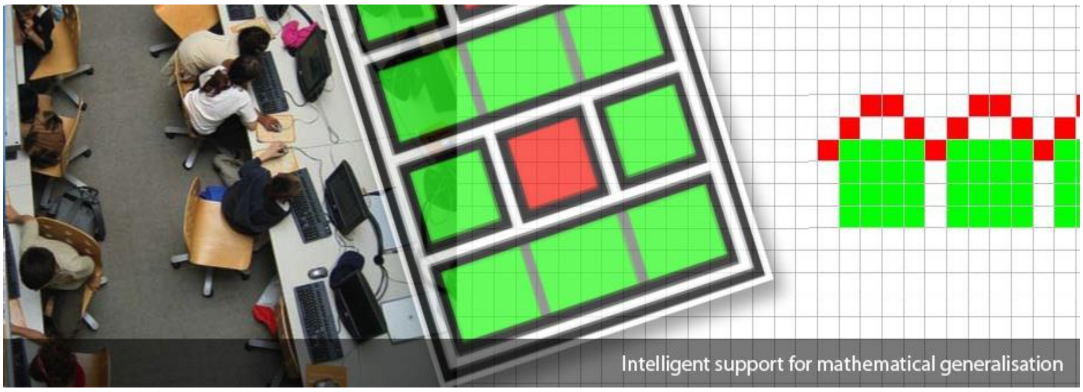

ABSTRACT

An exploratory learning environment for supporting the construction of mathematical generalisations & their expression as rules

migen intelligent support for mathematical generalisation

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Acknowledgements

We acknowledge with thanks the input and support of teachers and teacher educators in the development of the package

Introduction

The MiGen[1] project creates an exploratory learning environment that supports the construction of mathematical generalisations and their expressions as rules. It provides students with a basis for learning school algebra and developing algebraic skills. Computer software, the **eXpresser**, was designed to exploit animated models of pattern structure to express generality. Our research studies with teachers, students and student teachers, have steered the development of the software, its interface and the associated activities. In the final phase of the project, the team have packaged the software with the activities with teacher guidance and student examples.

The eXpresser activities and the software are designed for use in Key Stage 3 classrooms. To assist teachers, there are online tutorials, online videos which model teaching approaches, as well as teachers' notes that accompany each activity.

Students can also build computer models in order to address some of the conceptual challenges of algebra. All the activities are aligned to the Key Stage 3, National Curriculum.

Pilot studies showed that students benefited when lessons using **eXpresser** were interposed with offline lessons where the rich teaching points that emerged from the student/computer interaction could be shared and consolidated in classroom discussion. This pattern of classroom use also allowed teachers to maintain a overarching learning trajectory for the class, while simultaneously assessing individual learning. The activities in this package helped "bridge" students' move from imagery or natural-language explanation to formal algebra. It provides for example, a context for collecting like terms, discussing the equivalence, of algebraic expressions.

We have included in the package a sample scheme of work and teaching sequence, used by one of the teachers with whom we worked, along with some examples of students' work.

eXpresser is available online at: http://expresser.lkl.ac.uk

A video explanation of the MiGen Project can be found here:

http://www.youtube.com/watch?v=ePq1qmPqgCY&list=UUYjqfZgaZgqZSHRQJ8qghfQ

[1] The MiGen project was funded by the ESRC/EPSRC Teaching and Learning Research Programme (Technology Enhanced Learning; Award no: RES-139-25-0381) and the MiGen Follow-on project is funded by ESRC, Award no: ES/J02077X/1

National Curriculum Links

The new Mathematics Programme of Study for Key Stage 3 (available at https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study) is now taught from September 2014. It includes in its aims that students:

- "….develop *conceptual understanding…"*
- "reason mathematically by ….*conjecturing relationships and generalisations…and developing an argument* using mathematical language*"*
- "solve problems ...by breaking down problems into a series of simpler steps *and persevering in seeking* solutions".

Students should also be able to "move fluently *between representations* of mathematical ideas" (emphasis added).

In terms of conceptual understanding and ''higher order" thinking as it relates to the development of algebraic understanding, the Programme of Study states that pupils should be taught to:

- "use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships"
- "freely move between different numerical, algebraic … and diagrammatic representations"
- "develop algebraic…fluency"
- "make connections between number relationships, and their algebraic representations"
- "make … conjectures about patterns and relationships"
- "begin to reason deductively in … algebra"
- "interpret mathematical relationships ... algebraically"

The activities using eXpresser are aligned with all of the above Programme of Study.

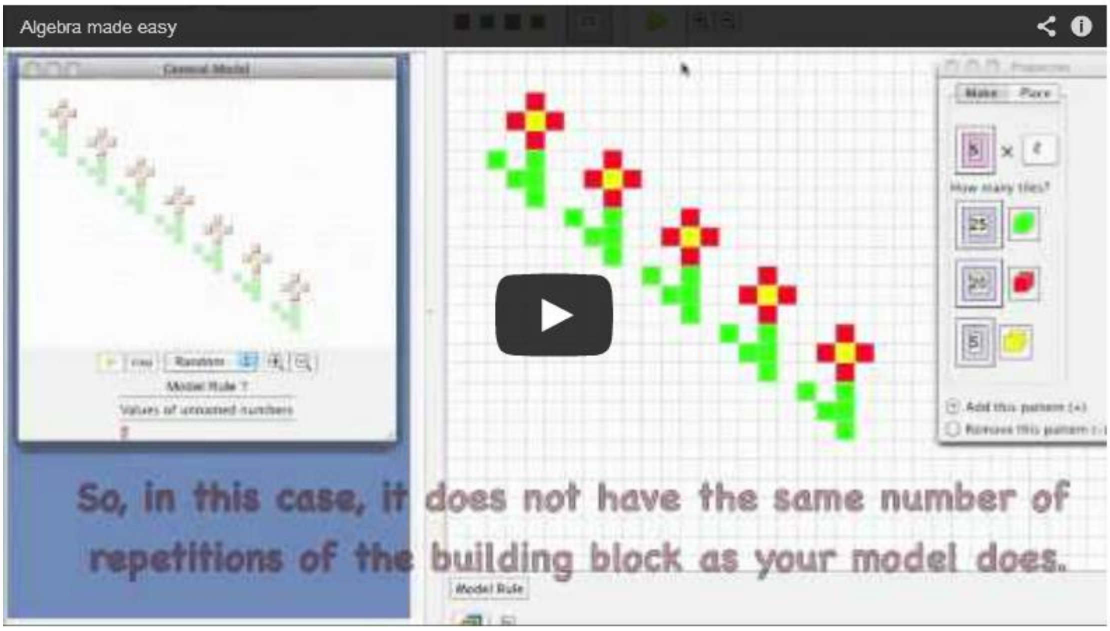
The algebraic skills developed through use of the package include the following bullet points from the Programme of Study for algebraic skill development:

- "substitute values in expressions"
- "identify variables"
- "express relations between variables algebraically"
- "use the vocabulary of expressions, equations, terms and factors"
- "model situations by translating them into algebraic expressions"
- "simplify and manipulate algebraic expressions to maintain equivalence"
- "generate terms of a sequence"

Tutorials

The main tutorial video is available in the ‘About’ section of the website with the eXpresser activities:

Tutorial Video: http://expresser.lkl.ac.uk/about.html



There are also several online tutorials around specific eXpresser tasks:

Tutorial 1 – Playing with numbers and expressions
http://web-expresser.appspot.com/?contextKey=new&tutorialId=1

Tutorial 2 – Making and animating simple patterns
http://web-expresser.appspot.com/?contextKey=new&tutorialId=2

Tutorial 3 – Colouring Patterns
http://web-expresser.appspot.com/?contextKey=new&tutorialId=3

Teachers’ Notes

There are notes throughout the package to refer teachers to questions and difficulties that students may have, teaching points that arose in trialling the activities, questions that can be posed to encourage teaching points, as well as guidance on using the software. The notes also include links between activities and suggestions for consolidation work.

The package also has reference guides and worksheets for each activity. These variously draw students’ attention to features of eXpresser that can be deployed to tackle the activity, offer suggestions for a way in to an activity as well as challenges and extension questions.

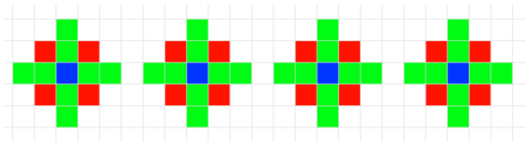
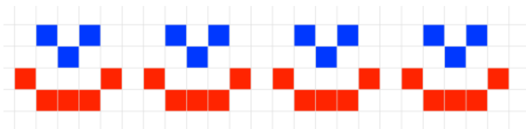
The eXpresser activities are principally intended for classroom-based work but can be readily adapted for homework or, with appropriate introduction, for independent work. The activities should be used in conjunction with the Framework and the Feedback Summary on page 7.

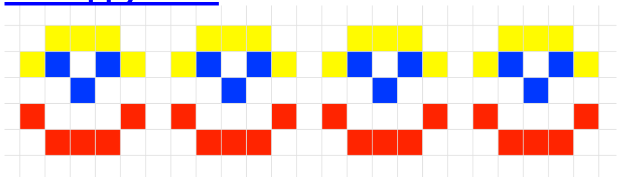
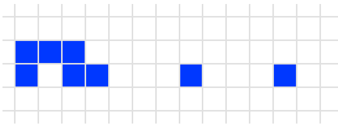
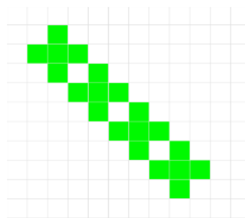
Teachers have commented on how their teaching was improved following the use of the package. They reported that they had gained greater insight into how students’ algebraic thinking skills developed and how eXpresser provided an interim language for describing models and a meaningful context for formal algebra. In the table below, we present some terms designed for and used in eXpresser and their analogue in algebra.

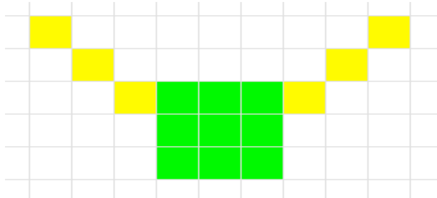
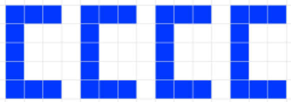
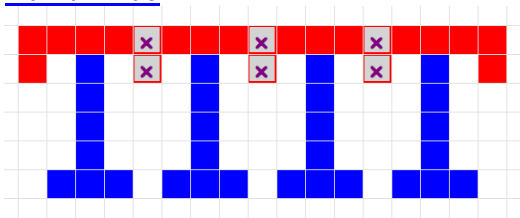
EXPRESSER	**ALGEBRA**
an unlocked number	variable
a locked number	constant
building block	a repeated element of a pattern
model rule	expression

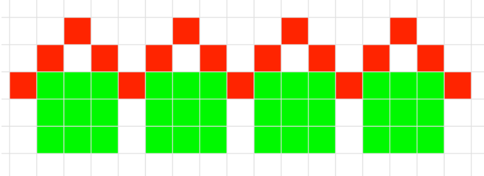
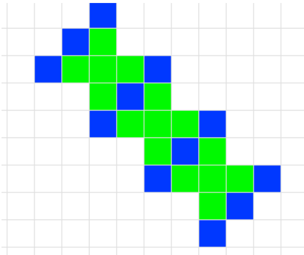
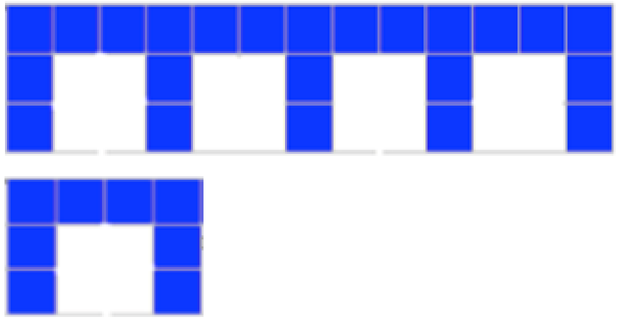
Framework for Activities

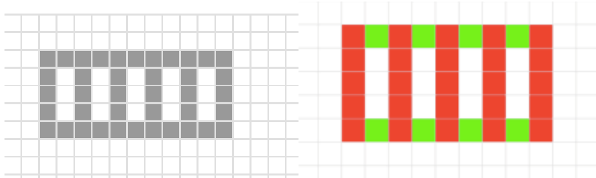
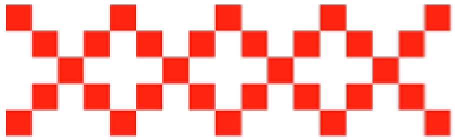
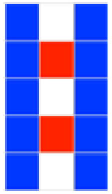
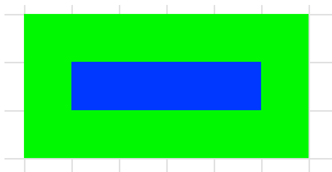
This framework locates each activity in the context of **eXpresser** tools, algebraic learning and the Key Stage 3 National Curriculum.

Focus	**Notes**	**Activity**	**Teaching notes and examples of corresponding algebra**
Using eXpresser	Exploring design features of the eXpresser: Building models in eXpresser requires creating building blocks to produce a pattern. The computer is then given a rule to describe the pattern of the building block (first by specifying the number of repetitions, then by using a variable for “any number”). With this rule the pattern can then be animated, and reproduced for any number of repetitions.	*Using the eXpresser to make patterns.* **Stars** 	Introduction to using eXpresser.
Generalising	Moving from a specific to a varying number of repetitions of a pattern is achieved by “unlocking” a number and giving the unlocked number, a name. At this point a slider appears by which means the value of the now named variable can be changed (within a specified domain).	*Making patterns of one building block.* **Mr Happy** 	Enrich early experiences of unlocking by highlighting students’ use of slider to see how the pattern changes once a number is unlocked. $y = mx$

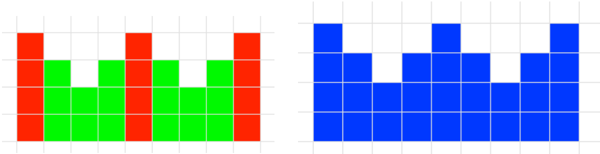
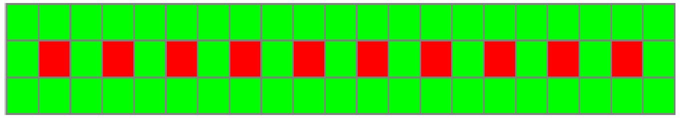
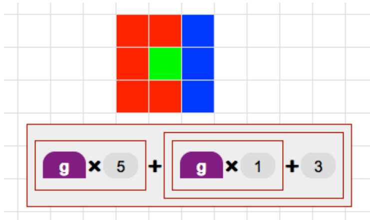
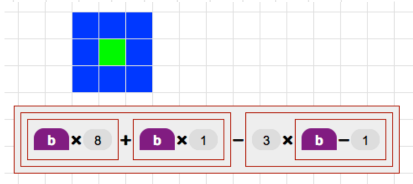
Generalising	If the structure of the pattern is received as two building blocks these have to be created separately and linked.	Activities can involve more than one building block *Adding a building block to a pattern* **Mr Happy's Hat** 	Total number of tiles = 5x +3x +5x x is the model number
Operating on variables	Solving the problem of two unlocked numbers.	*Two unlocked numbers* **Help Tim**  	Total number of tiles = x+5(x+1)
Pattern Placement	The eXpresser defaults to four horizontal repetitions when a pattern is created. **Crosses** shows how to change this.	*Changing horizontal repetition* **Crosses** *Develop pattern placement* 	y = 5x

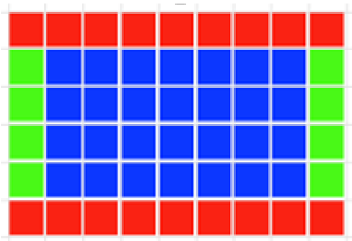
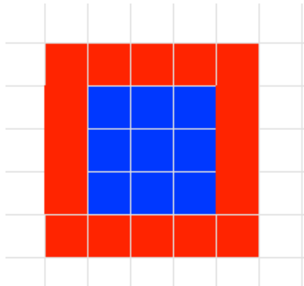
	Reindeer develops pattern placement – one antler is “broken” and the model needs to be realigned to animate correctly.	**Reindeer** 	Y = 3x + 3x + 9
Negative Patterns	Removing parts of a pattern that overlap is done in eXpresser by creating a pattern that is then “removed”. (This also demonstrates equivalence between adding a negative and subtracting a positive, and additive inverse.) **Powerlines** is a task to fix the overlapping problem, giving practice in using negative patterns. The **Houses** model can be built using a negative pattern, or by adding a constant	*Removing parts of pattern* **Changing Letters**  to  *Negative patterns* **Powerlines**  *Comparison and equivalance*	y = 10x – x y = 14x – 2(x-1) y = 5x + 9x - (x - 1)

	term. This invites a discussion of comparison and equivalence.	**Houses** 	
Operating on variables; linking patterns	Patterns with multiple building blocks require expressing the number of one building block in terms of the number of another. Expressions are needed to describe the connection between the building blocks, as a way to describe the relationship between variables.	**Lines and crosses** 	y = 5x + 3(x + 1)
Constant terms	Constant terms are introduced by adding a building block that does not have an unlocked number.	**Bridges** 	y = 5x + 3 or y = 8x – 3 (x – 1)

Equivalence	What is equivalence and how is it established for example by evaluating certain terms, or by looking at structure? In eXpresser, structure is analysed using building blocks. Colour is used to encourage the identification of the structure of a pattern. As familiarity with eXpresser develops, colour becomes less a feature of the building blocks, and more a means to describe how the pattern is structured. In this way, colour acts as a scaffolding mechanism, initially by aiding the identification of the building blocks to describe structure, and subsequently as a means to assist the comparison of different structures and assess equivalence.	**[Traintracks]**  **[Stars]**  **Ladder** 	Most tasks can be used to encourage awareness of equivalence. Tasks can be presented without colour and then students are encouraged to colour in different ways to come up with different models and rules y = 7x + 5 (Traintracks) y = 7x + 2 (Stars) y = 4x + x + 2 (Ladder)
Dependent variables	Tasks where a base model is given and a structure is required to be superimposed call for an analysis of how different but constant variation works when one or more aspects of the variation are given. Dependence and independence relationships can start to be identified. In this task, a blue 'pond' has to be surrounded by a green path.	**Garden Pond** 	y = 2x + 6

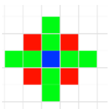
OTHER CHALLENGES AND IDEAS USING EXPRESSER

Focus	Notes	Activity	Teaching notes and corresponding algebra
Growth and direction	Creating a model that animates in multiple directions	**Growing Out** 	Start with one tile, and set the challenge to make a model, which "grows" out in 2, 3 or 4 directions. Y = 3x + 1
	What happens if the domain of the variable is restricted?	***Skyscraper** 	y = 12x + 4 (x has min value of 1) or y = 4x + 8(x-1) but (x has min value of 2)
Differences in structure	Colour is used in this activity to differentiate differences in structure	**Footpath**  Colour is used in this activity to differentiate differences in structure. If the building block is identified as a column (or an "I"), then a different model rule is produced than if "C" or a "D" is used.	 y=5x+x+3  y=8x+x-3(x-1)

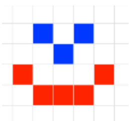
Multiple variables	Width can be given as a starting block	**Pool – Rectangle** 	Total number of tiles y = 2(h + w) + 4 (+ hw)
Quadratics		**Pool – Square** 	The path around a square pool gives 4x +4 tiles The total number of tiles for the pool and the path is (X^2 + 4x +4), which can be factorised to $(x+2)^2$

INTRODUCTORY ACTIVITIES

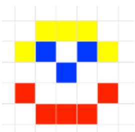
These are step-by-step activities to introduce how the eXpresser works. We have 3 activities:



STARS (creating a model)



MR HAPPY (animating a model)



MR HAPPY's HAT (linking patterns)

Activity 1 - Stars

Title	Stars
eXpresser Objectives	Gain familiarity with eXpresser
Mathematical Objectives	• Encourage the exploration of the structure of patterns • Create a pattern • Identify elements of structure • Create patterns for different repetitions
Teacher Notes	This activity is done on the Free Play area of the eXpresser, and is a simple guided exercise through the process of building a pattern. It can be done step by step, with a teacher modelling each step and each member of the class then creating the building block. Doing the exercise as a class activity to introduce eXpresser worked well when a student was selected to perform the steps as the rest of the class read out the instructions. The teacher can then pause to highlight features and draw attention to feedback that the system gives (coloured animation, smiles etc.), allowing time for students to write their answers to the questions.

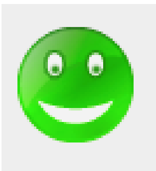
Task / Activity

The aim of this activity is to become familiar with the features and the language of the eXpresser. This activity will involve creating a pattern, identifying elements of its structure, and working on how to find a rule to recreate the pattern for different numbers of repetitions. Later activities will focus on generalising the rules.

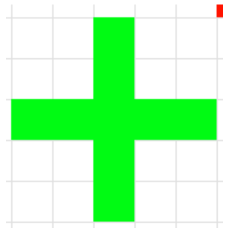
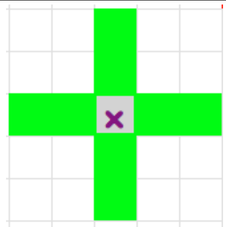
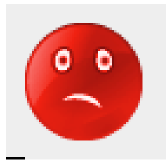
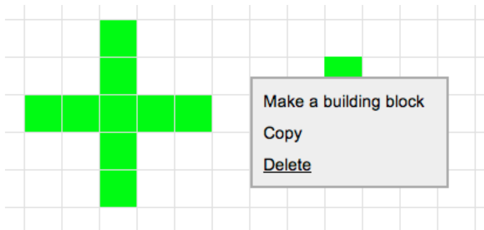
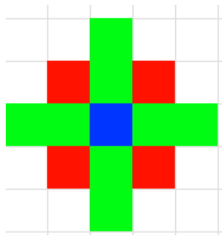
By the end of the activity we will have a pattern that is coloured for a specific number of repetitions.

In this activity, we will:

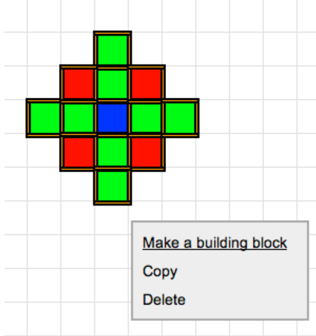
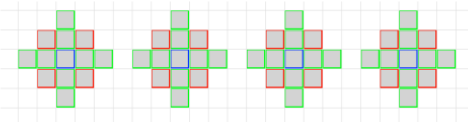
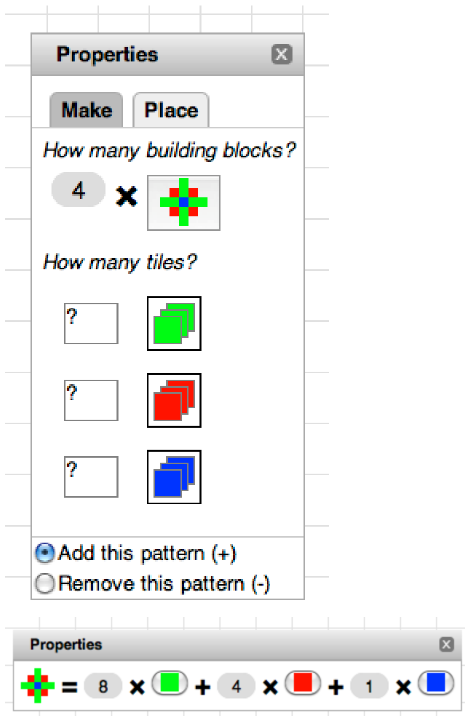
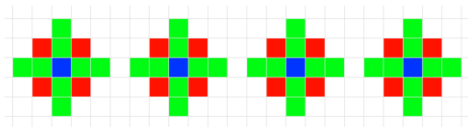
- move tiles around the canvas by clicking and dragging;
- delete tiles from a pattern;
- create a building block;
- create a pattern and identify its properties;
- try varying the number of terms in the pattern to look at what happens; and
- make Smiley happy.



Making a pattern

1. Create a green cross that is 5 tiles across and 5 down by clicking on the green tile icon and dragging green tiles to the white grid (this is the canvas). How many tiles have you used to make your cross?	
2. Click and drag another tile to the centre of the cross. Write down two things that happen? *(a) the centre tile loses its colour, (b) the centre tile is replaced with a cross, and (c) Smiley is unhappy*	 
3. To remove the extra tile click and drag it to a blank part of the canvas. Now delete it: left click and choose "Delete". **Notice that all actions with the eXpresser are done by *left* clicking**.	
4. Replace the green tile at the centre of the cross with a blue tile. **(You can delete the green tile directly from the centre, but it is often easier to drag things to the blank canvas and work on things there.**) Don't forget to delete the green tile. (Left click and choose "Delete".)	
5. Now click and drag 4 red tiles to your cross to make a star. Your star should look like this.	

Creating a building block and a pattern

6. Select the star by left clicking and dragging over the whole shape. Select "Make a Building Block"	
7. Click on the star, and select "Make a pattern". By default, the eXpresser chooses to repeat the Building Block 4 times in the horizontal direction. Click OK.	
8. We want the pattern to be coloured. To colour the pattern, put the number of tiles of each colour in the right box. Click on the star in the dialogue box to reopen the Properties window. This will show you what you need for each *building block*. How many of each coloured tile do you need for the *pattern* of 4 building blocks? Green: _____ Red: _____ Blue: _____ Put the number for the red tiles into the number generator box at the top of the screen and drag the number to the box with the question mark. You can release that number when the frame of the box with the question mark gets highlighted with a red line. Now do this for each other colour.	
11. What happens when you do this? *The pattern is coloured.*	

12. Change the number of building blocks in the pattern to 3: put 3 into the number generator box and drag it over the 4 in the pattern dialogue box, then select 3 from the drop down box.

(This may take a bit of practice, so just remember to right click to delete anything you don't need).

The pattern is not coloured.

Put the correct values for the number of tiles needed so that the pattern is coloured again.

Write the number of coloured tiles in the spaces below:

Green: ______ Red: ______ Blue: ______

This will help you get used to how to change inputs and select what you want from the menu.

Can you see a connection between the number of green tiles in one building block and the number of green tiles in the pattern? What is the rule for how many green tiles you need?

How many tiles would you need if there were 5 building blocks in the pattern?

Green: ______ Red: ______ Blue: ______

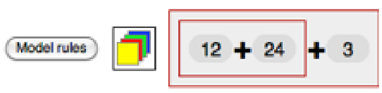
Now check you are right by seeing if the pattern is coloured when you put your answers to the properties box.

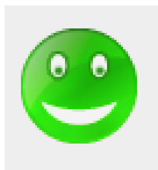
14. Can you work out the rule that connects the **total** number of tiles to the number of stars?

To finish this activity, let's make Smiley happy. He wants to know the total number of tiles needed to make the pattern.

One by one, click and drag the number of tiles of each colour to the "Model Rules" box at the bottom of the screen. This should give you 12 + 24 + 3.

In the next activity we will look at how to give the eXpresser those rules so that the pattern is coloured for any number of building blocks.





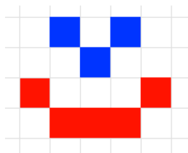
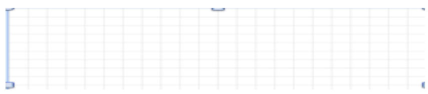
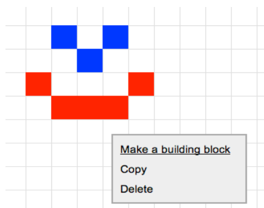
Activity 2 – Mr Happy

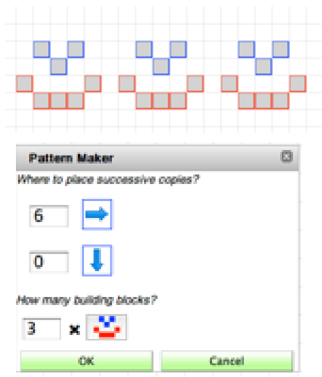
Title	Mr Happy
eXpresser Objectives	Creating building blocks, creating and animating patterns, producing a model rule
Mathematical Objective	• Making a variable
Teacher Notes	This is a guided "how to" activity where students create and animate a model with one building block. You may want to have students watch a demonstration where a simple model is created and animated and a model rule is given prior to starting the activity, if possible, or some teachers have found making the Mr Happy model together as a class, helps students by allowing them to see how the eXpresser works generally, and provide an initial guide to its layout. In trials, teachers found a detailed guide to the activity valuable at this early stage, as students can then move at their own pace through the steps of model making. It is also a useful reference for later lessons when students have forgotten the steps to animating the model. Having a paper record of the students' work was also considered helpful for consolidation and reflection away from the computer.

<u>Task/Activity</u>

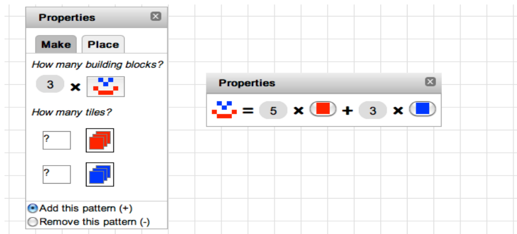
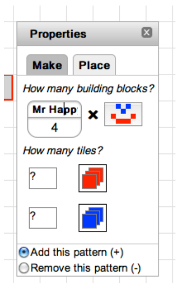
The aim of this activity is to create a model that the eXpresser can animate. We will make a pattern that remains coloured for any number of repetitions and that animates in the eXpresser.

Creating a pattern

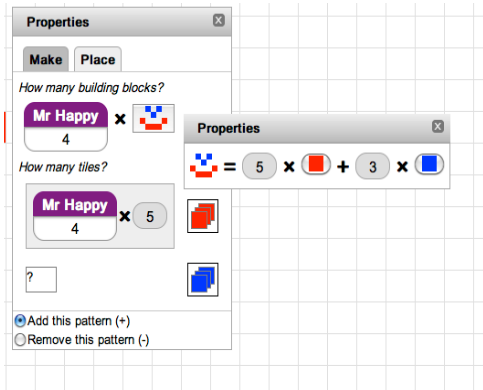
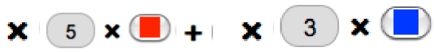
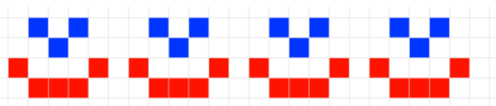
1. Launch the activity. Create a face by clicking and dragging red and blue tiles to the working area.	
2. Draw your building block.	 How many **red** tiles? _____ How many **blue** tiles? _____
3. Complete the rule for one building block.	_____ ×  + _____ × 
4. Make a building block by selecting the whole picture, left click and select "create a building block" from the menu.	

5. Create a pattern by selecting the whole picture, left click and select “create a pattern” from the menu. Click OK.	

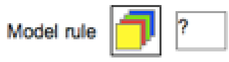
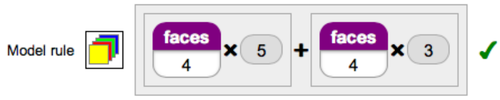
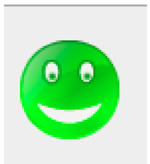
Unlocking numbers

1. Click on the picture of the face in the Properties box to get the rule for the number of tiles	
For four copies of the building block: Complete the rule for **4** building blocks:	How many red tiles? _____ How many blue tiles? _____ _____ ×  + ____ × 
2. “Unlock” the number of faces: Click on this number and select “Unlock”. Give your unlocked number a name.	
What is the name of your unlocked number?	
3. The slider appears at the top of the screen. This allows you to change the number of faces in the pattern.	

Creating expressions

1. To colour the pattern you need to write a rule for the number of tiles of each colour. On the working area, create an expression for the number of red tiles by clicking and dragging the 5 (from the number of building blocks), and clicking and dragging the unlocked number to hover over the 5, drop it and then connect them with a multiplication sign. 2. Repeat for the number of blue tiles.	
Use the slider to change the number of faces in your pattern. Write the rule for **6** faces.	Red Tiles ____  Blue Tiles ____  Total Tiles __ 
3. Click and drag the expressions to the box asking for the number of tiles.	
4. Check using the slider that the pattern remains coloured for any value of the unlocked number.	

Giving the model rule

1. To get a smiley, you need to give the computer the rule for the total number of tiles in the model.	
2. Click and drag your blue tile expression to the box with the question mark at the bottom of the working area. When the box is red, drop the expression. Do the same with the expression for the blue tiles. Have you got a tick and a smiley?	 

Animating the General Model

1. To check if you have completed the task successfully, you need to animate the General Model to see if it stays coloured. Click on the General Model icon at the top of the screen. Move the grey vertical bar that separates the 2 'windows' in eXpresser to the right for more space. Now click on the arrow to animate the model.	 

Using the Model Rule

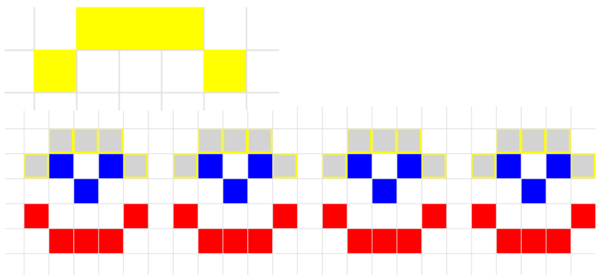
Use the slider to answer these questions. (Remember you can click on expressions or model rules and select "calculate value" to find the number of tiles.) Show your working (write down the expression or rule that you use to get your answer).	How many **red** tiles are there in Model Number 10 (10 faces)? ____________________ How many **blue** tiles are there in Model Number 15 (15 faces)? ____________________ What is the total number of tiles in Model Number 8 (8 faces)? ____________________

Activity 3 – Mr Happy's Hat

Title	Mr Happy's Hat
eXpresser Objectives	Linking patterns, practice with creating building blocks, creating and animating patterns and producing a model rule
Mathematical Objectives	• Making and linking variables
Teacher Notes	This is a guided "how to" activity aimed at linking patterns, a step which is needed for models made out of more than one building block. The activity starts with the Happy Faces pattern already made. The aim for students is to recognise the need to use the unlocked number from the Mr Happy pattern for the Mr Happy's Hat pattern; if a second unlocked number is used, the number of hats will not be linked to the number of faces that is the number of hats is dependent upon the number of faces. Try to help students answer this for themselves. To practise linking models, get students to create a further pattern and add it to the model such as putting a green pompom on top of the hat, or giving Mr happy a beard.

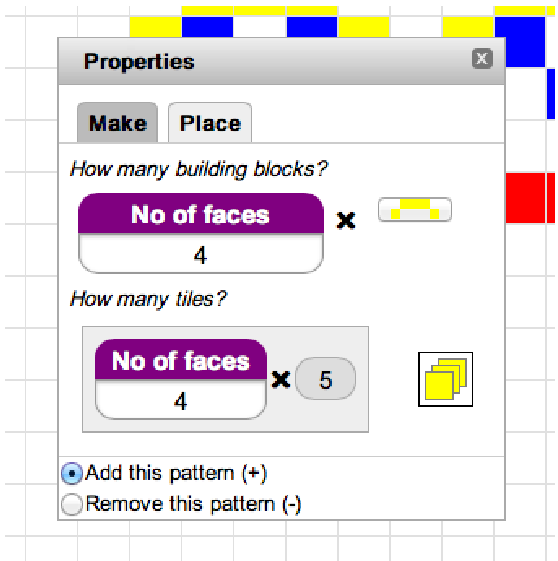
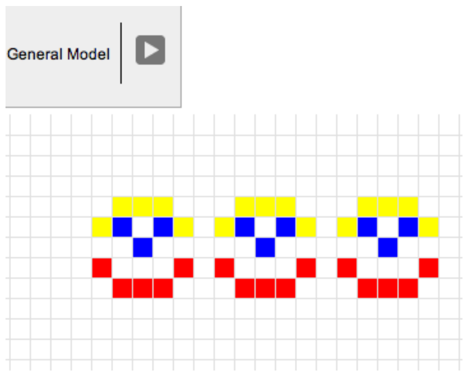
<u>Task/Activity</u>

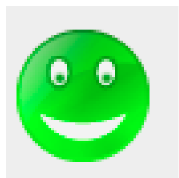
The aim of this activity is to link one pattern to another to create a model. We are going to add a hat to the model of Mr Happy.

1. Launch the activity on the website. Create the hat pattern by clicking and dragging yellow tiles. (You can create a pattern anywhere on the working area and then drag it where you want to.)	

<table>
<tr>
<td>2. To colour the hair you need to create an expression for the number of yellow tiles.

Since you need to put hair on each face in the model, the number of hats must equal the number of faces. To do this, you have to link the number of faces to the hat pattern, by using the same unlocked number for each pattern.

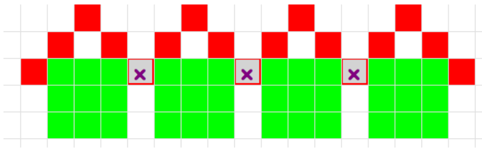
Click on the Properties box of the Mr Happy pattern and drag the unlocked number to the expression you are creating for the hat pattern.</td>
<td>
</td>
</tr>
<tr>
<td>3. Animate the pattern in the General Model to make sure it remains coloured.</td>
<td>
</td>
</tr>
<tr>
<td>4. Remember to give the model rule, and get your Smiley!</td>
<td>

</td>
</tr>
<tr>
<td>5. Challenge:

Can you put a pompom on the hat? Make sure you update the model rule to get Smiley happy again.</td>
<td></td>
</tr>
</table>

MAIN ACTIVITIES

If you try to put more than one tile on a square on the eXpresser working area an “X” appears, because each square can only have one tile on it. When you create models in the eXpresser using more than one pattern, sometimes parts of the patterns overlap, like this:



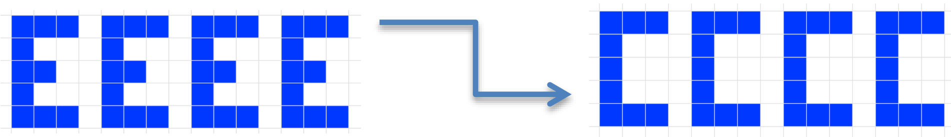
If you want to remove a part of a pattern because it overlaps, you need to **create another pattern,** which is exactly the same as the tiles you want to get rid of, and then tell the computer to ‘**remove’ that pattern from the model**. There are several metaphors to help think of this. Patterns that are removed from the model are in essence negative numbers and so can be called **negative patterns**. Some students called these patterns ‘eraser’ patterns. In the dog model above, we would need to create a pattern made up of just the tail of the dog, and then think about how many tails need to be removed (how many tails are overlapping). The challenge in these occasions is to think how to do this in a general way. In both patterns above the number of tiles to be removed is one less than the number of building blocks of the original pattern.

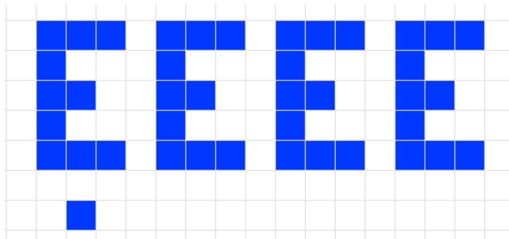
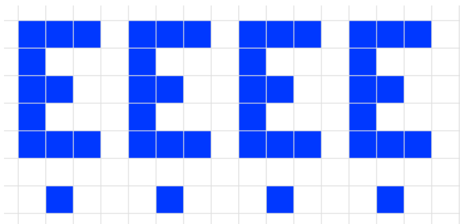
In the activity that follows, we will practice creating a new pattern and then tell the computer to remove it from the model, so we will end up with a new model. We will change E to C:

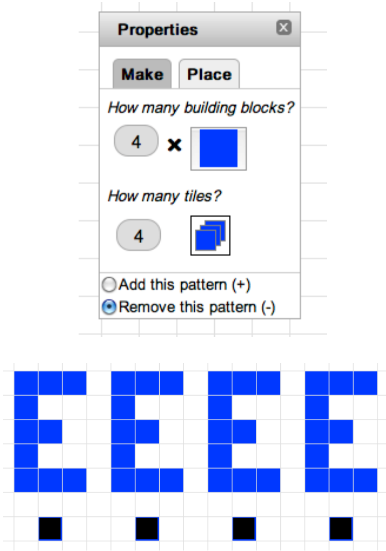
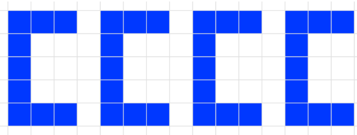
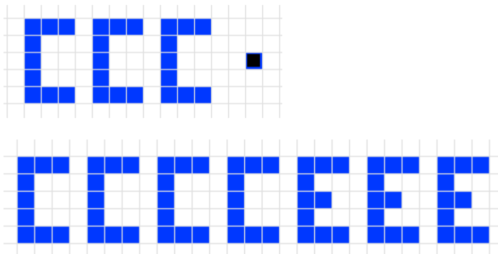
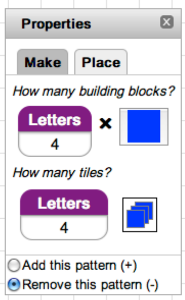
Theme – Building Negative Patterns

Activity 1 – Changing Letters

Title	Changing letters
eXpresser Objectives	Creating a new pattern and then removing it to create a new model
Mathematical Objectives	• Introducing negative numbers
Teacher Notes	Before leaving this activity, look carefully at the model rule in section 7:  Notice that it is showing a subtraction, but we have actually created a new pattern for our model! We turned it into a negative pattern when we turned it black, so in fact we added a negative pattern: **10 x Letters + -1 x Letters** But the eXpresser has created: **10 x Letters - 1 x Letters** This works because adding a negative number is the same as subtracting a positive number. Try it out for yourself!



1. To make the E into a C we need to remove the tile in the middle of the E, so we need to make a pattern that we can place over that tile. Create a building block of one blue tile. Place it below the E pattern.	
2. Now create a pattern from the blue building block, making sure it is placed exactly where the middle tile of all the E's in the model are.	

3. At the bottom of the Properties box for your new pattern, select “Remove this pattern”. This will turn your new pattern black.	
4. Now drag your new black pattern up to the middle tile of the E and drop it. What happens?	
5. Animate your model in the General Model. Are there E’s and C’s mixed up and sometimes black tiles like the examples shown in this figure?	
6. How can you make sure that for every E in your model there is one black tile? (You can do this by clicking and dragging the unlocked number on the slider to where you need it.)	
7. Give the model rule at the bottom of the eXpresser’s working area. Is Smiley happy? Well done!	

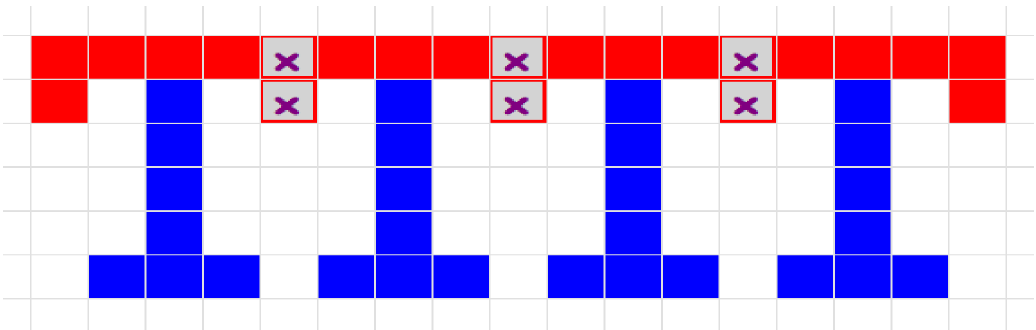
Theme – Overlapping Patterns

Activity 1 – Powerlines

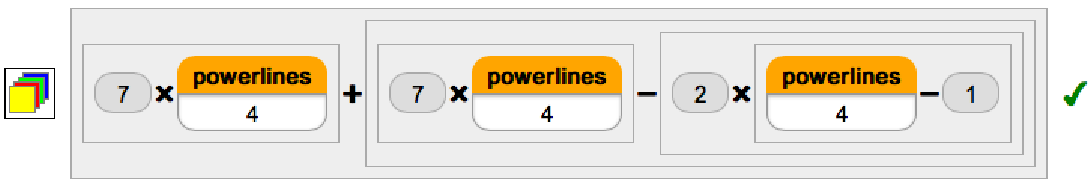
Title	Powerlines
eXpresser Objectives	linking patterns, using negative patterns
Mathematical Objectives	• use of the additive inverse • use of negative patterns
Teacher Notes	This activity follows from **CHANGING LETTERS** and develops the thinking behind subtracting parts of a pattern. It is presented as a model that needs fixing, so encourage students to work with the information that they have been given (by clicking on the model they will be able to see the properties of the powerlines model). The principle aim in this activity is to use **negative patterns**, by fixing up a pattern that has an overlap. A reminder of how to delete a tile in a pattern, either as a starter activity or once students have come up with a need to "get rid" of some tiles, may be useful. Students could redo the **CHANGING LETTERS** activity.

Task / Activity

The principle aim in this activity is to use **negative patterns**, by fixing up a pattern that has an overlap. A reminder of how to delete a tile in a pattern, either as a starter activity or once students have come up with a need to "get rid" of some tiles, may be useful. Students could redo the **CHANGING LETTERS** activity.



Note that the model rule shows the addition of a "removed" pattern as a subtraction. This is discussed in the notes to the **CHANGING LETTERS** activity. These activities may be useful starting points for a discussion away from the eXpresser environment about the use of the additive inverse, as well as illustrating how adding a negative number has the same effect as subtracting a positive number. This may, for example, help with revision on operations on negative numbers.



Students will need to find a rule to say that there is one less overlap than the number of powerlines (i.e. p-1 times the number of tiles in the overlap). Operating on variables is an important, but difficult, algebraic skill that students need. (See the notes to the **LINES AND CROSSES** activity for more on this.)

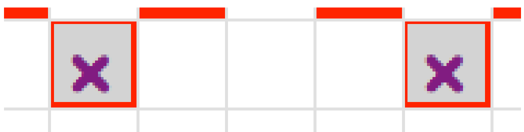
The aim of this activity is to use what you have learnt about adding and subtracting patterns to fix the model. Your task is to fix the model so that there are no overlapping powerlines.

To get started:

1. Remember you can only have **one** tile on a square. A cross on a tile means that there are too many tiles in that square. You will need to remove one.

2. How many patterns are the powerlines made with? Click on the powerlines to find the Properties box. This will give you some clues about how the powerlines are built.

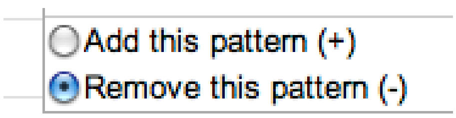
3. Is there a pattern for where the crosses are? In the example below there is one cross **every four tiles**:



4. Think about how to make a pattern that matches your crosses. Use the slider to check your thinking about the pattern – when the slider is on 4, how many overlapping tiles are there? What about when it is on 6?

5. How does the pattern of overlapping tiles change when the slider does?

6. Use the "Remove this pattern" option to take away a pattern:



(If you need a reminder about how to do this, go back and look at the **CHANGING LETTERS** activity.)

7. Remember that you can create patterns on the working area away from your main pattern. This can give you room to work out how you need to build your patterns.

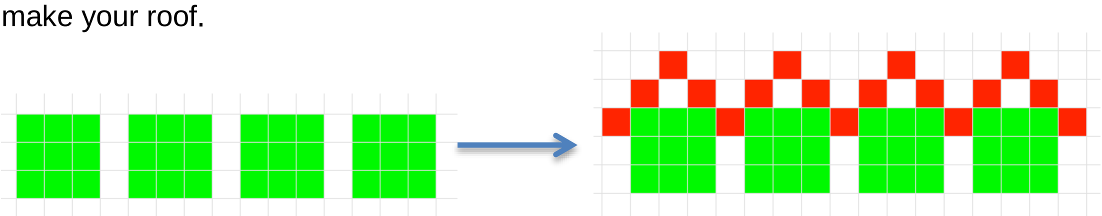
Activity 2 – Houses

Title	Houses
eXpresser Objectives	linking patterns, using negative tiling patterns
Mathematical Objectives	• confident use of constants and variables, • recognising equivalence of algebraic expressions, • use of the additive inverse, • how to use equivalent expressions
Teacher Notes	One of the aims of this activity is to develop a familiarity with using **negative patterns**. Trials showed a general preference for additive approaches to model building, as opposed to using negative patterns. Perhaps this is because subtraction can be considered harder than addition, or that the use of negative patterns is less "obvious" than creating a model additively. The task can be completed without using a negative pattern, so, if use of a negative pattern is the teaching/learning focus of the activity, insist that the roofs be built (at least the first time) using a negative pattern. There are a number of ways the model can be built, and this is always helpful to reinforce the inverse nature of addition and subtraction, and "choosing" what suits a particular task. Allowing students to find and then compare their preferred view of the structure of the pattern is an important aspect of learning with the eXpresser because it develops the idea that structure can be seen differently, and exploring those differences requires a language of description. The eXpresser terminology is deliberately construction–based ("building blocks, "make", "place", etc.). These, and the use of coloured tiles, allow discussion and comparison, and are all precursors to the more formal algebraic terminology architecture of terms, constants, brackets, co-efficients and expressions, etc.

<u>Task / Activity</u>

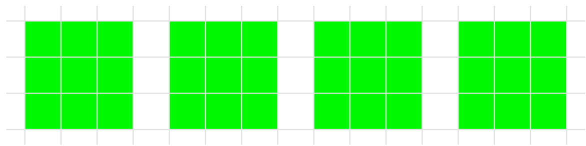
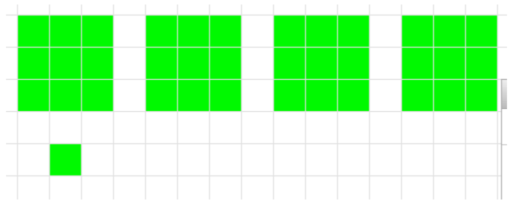
The aim of this activity is to link patterns. One pattern has been created for you, and your task is to build the required model by adding patterns. You will also need to think about how to take away parts of a pattern if there is an overlap.

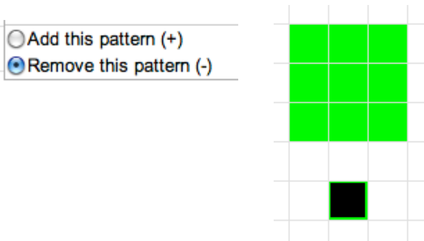
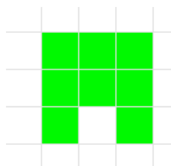
You need to add a roof to the row of houses. You can use any colour or colours to make your roof.



Start students off on the structure of the roof – build one as a building block then see what happens when they make a pattern with it. It takes time to think through what is happening, so students may need some time in working out why they cannot just repeat the first roof, and how that might be able to be fixed using another pattern.

A reminder of how to delete a tile in a pattern, either as a starter activity or once students have come up with a need to “get rid” of some tiles, may be useful. One suggestion follows, with the students either watching or doing it themselves:

1. Launch the activity.	
2. Before starting the roof exercise, suggest that we build a door first: create a one squared building block and pattern.	

3. Don’t unlock any numbers; the intention here is to start the thinking. “Remove” the pattern.	
4. Drag the “negative door” pattern and drop it over the tile for the appropriate tile in the house. The result is a blank space.	

This exercise provides a reminder of negative patterns. We have created a pattern specifically to delete one tile in the 3x3 square building block (i.e. 1 + -1 = 0), which produced a door in the house. In the task, a pattern needs to be created to remove the overlap, so that the tile between the houses is coloured and not marked with a cross (i.e. 2 + -1 = 1). This may need to be made explicit.

To get started:

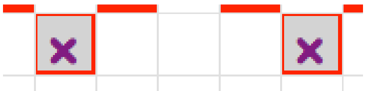
1. Start by building the roof for the first house. Click on the house to get the Properties box. This will give you some clues about how the house pattern is built. See what you can use for the roof pattern.

2. Remember that you can create patterns on the working area away from your main pattern. This can give you room to work out how you need to build your model.

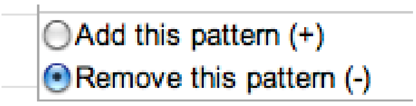
3. You can only have one tile on a square. If you get a square on your canvas that has a cross on it, that means there are too many tiles in that square. You will need to remove it (or them).

4. Is there a pattern to when you get a cross?

   Here it happens every 4 tiles:

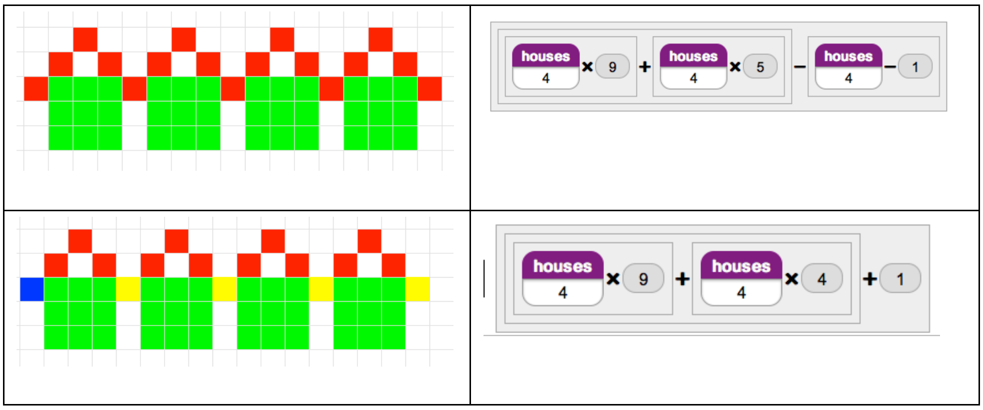


5. Think about how to make a pattern that matches your crosses and then taking it away using the “Remove this pattern” option:



(If you need a reminder about how to do this, go back to the **CHANGING LETTERS**)

Two solutions to the activity are:



Some further points that may be taken up are set out below:

- The model rules show how the model is built– each term in each of the rules refers to a colour in the model. Can students identify which term goes with which colour? Get students to “tell the story” of each solution from the model rule – this provides a rationale for using brackets, and is a good starting point for creating expressions.

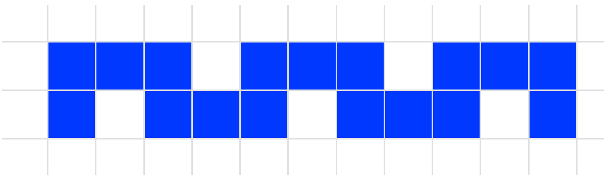
- How does the model rule show the instruction to “take away one less than the number of houses” in the first solution? (Another way of getting to the same solution is take away as many as the number of houses and then add one back; this illustrates that subtracting a negative number has the same effect as adding a positive number, and why we have the rule about two minus’s making a plus.

- The first and second solutions look similar – ask the class what is the same and what is different about them? The differences in each of the model rules reflect a different construction. Colour has been used here to show how the different models have been created. You might want to refer to the rules in formal algebraic notation and ask students to justify their equivalence.

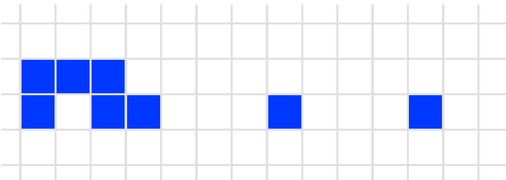
Theme – Linking Unlocked Numbers

Activity 1 – Help Tim

Title	Help Tim
eXpresser Objectives	Solving the problem of two unlocked numbers
Mathematical Objectives	• Creating and linking variables
Teacher Notes	This activity is presented with two patterns, each with an unlocked number. If the sliders are set manually then the model appears to work, i.e. with an "arch" at the beginning and the end. Students often unlock several numbers and struggle with how to connect the number of one building block, or pattern with another. This activity is designed to make this connection explicit by considering how the number of "arches" and "links" is related. This is done by examining the changes to each of the patterns that make up the model, and then looking at how the number of one pattern is linked to the other.



In this activity, you need to help Tim. He has used two patterns to create this model. The problem is that when he animates in the General Model, it messes up and it does not work the way he wants it to, with a column at the beginning and end:



1. Launch the activity.

2. Animate the pattern in the General Model and look at what is happening. Write a sentence to describe what you notice:

____

____

____

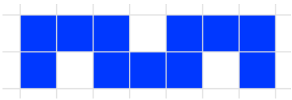
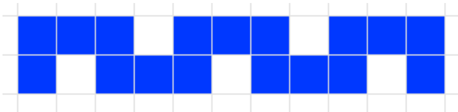
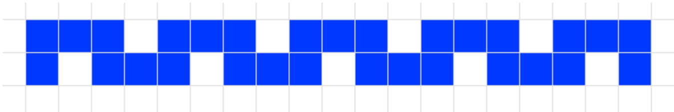
3. Now use the sliders. What happens if both sliders are set at 3? What if they are both set at 2? Write a sentence to describe HOW the model is not correct.

____

____

____

4. Write in the table below the setting for each slider that is needed to make the model correct:

Model	arch	link
		
		
		

5. Decide which of these statements is correct:

Look at the MODEL:	**True or false**
There is a **link** between each **arch**.	
There is an **arch** between each **link**.	
Look at the TABLE or THE SLIDER:	
The number of **links** is one less than the number of **arches**.	
The number of **arches** is one less than the number of **links**.	
The number of **links** is one more than the number of **arches**.	
The number of **arches** is one more than the number of **links**.	

6. How would you describe the connection between the values of the sliders when they are set so that the model works?

____

____

____

7. What is the rule for the number of tiles in the arch pattern (Look at the Properties box)?

____

8. What is the rule for the number of tiles in the link pattern?

____

____

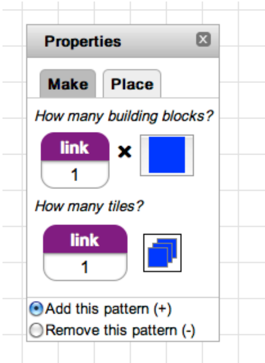
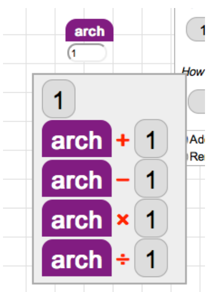
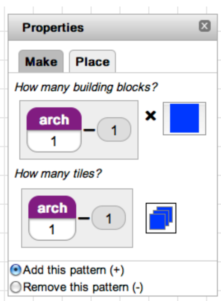
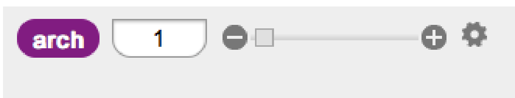
9. Now, circle the correct statement:

link arch = + 1	link arch = - 1

Check your answer by setting the arch slider to 6. How many links do you need? 7 or 5? Does this fit your rule?

---

10. We only need one slider, because in this model the number of links will always be one more than the number of arches. Follow the steps below:

STEP	ACTION
Left click in the Properties box of the Link pattern to relock it. link	
Create an expression for the number of links by dragging 'arch' and '1' to the canvas. arch 1	
Drag your expression to the Properties box:	
Set the slider to 1. What is the smallest value that 'arch' can take for the model to work? ______ Click on the settings button and adjust the limits.	

11. Now animate the General Model. Have you fixed Tim's problem? Does the model start and finish with a vertical column?

12. Finish the activity by giving the Model Rule (make sure Smiley is happy).

    Write your Model Rule here:

EXTENSION: Can you find the model rule using link ?

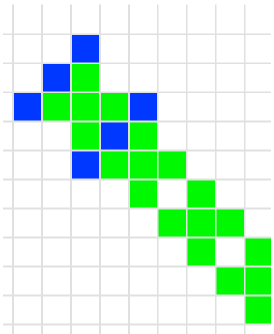
(Hint: if link = arch - 1, then arch =??)

Activity 2 – Lines and Crosses

Title	Lines and Crosses
eXpresser Objectives	linking patterns, unlocking numbers
Mathematical Objectives	• operations on variables, • use constant and variable terms, • recognise equivalence of algebraic expressions
Teacher Notes	This activity can be done as a standalone activity, or as a follow on from **CROSSES**. **CROSSES** looks at placement of patterns and how to vary the horizontal placement that the eXpresser defaults to. This exercise is designed to encourage students to link the use of variables, and to consider how one pattern varies with another. It is useful to do the activity beforehand, perhaps with colleagues, to see what structure or structures you "see" and work with, as there are a number of approaches that can be made. The diagonal positioning of this pattern presents an initial challenge (and the **CROSSES** activity deals with this aspect explicitly), as the horizontal/vertical placement needs to be varied. Students will need time to try various combinations. If they get stuck, consider prompting them to look carefully at the "Place" tab in the properties box of the crosses pattern, and use that information. There are some notes below about suggestions on how you might wish to support students' understanding of unlocked numbers, eXpresser's metaphor on variables. Students often unlock a second number for the blue lines. This is a logical step, and follows the idea implicit in the name of the unlocked number in the crosses pattern ("number of crosses"). Reflecting on what happens when this is done is useful, as is noticing eXpresser's feedback system, and how the model will "mess up". The screenshot below shows a second unlocked number called "no of lines". Expressing the number of lines using the number of crosses is not intuitive, and takes time to become apparent.

Task /Activity

If/Once a second number is unlocked, a rule can be given to the blue pattern to get it coloured (remember that colouring a pattern is part of the feedback system – colour means that the rule is correct for that specific value given to the unlocked number). Encourage the use of the sliders to explore when the model "works" and remains coloured. Students may think they have completed the task when they find one instance that "works", i.e. when there is a blue line at either end of the model. Animating the General Model will show that the activity is not complete, because the pattern "messes up" when the General Model is animated:

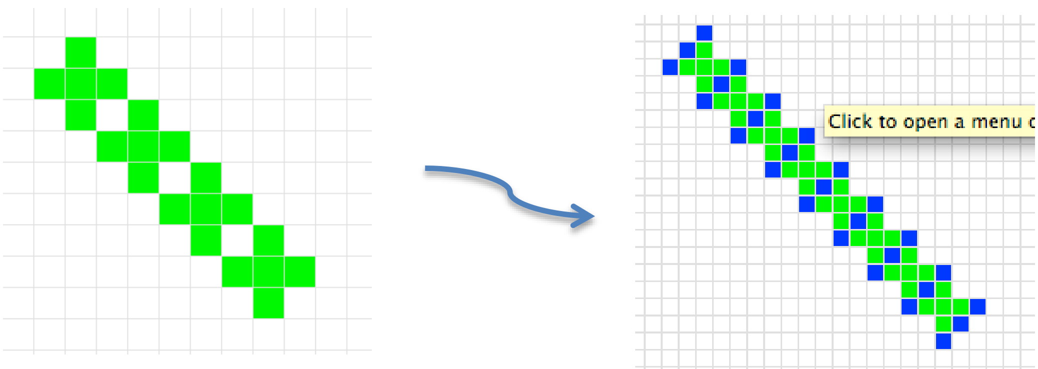


This means they have found that their solution holds for a specific instance, and now they need to work on the general case: what other instances can they find when the model "behaves"? Encourage them to "say what they see", so that the specific (such as "3 lines and 2 crosses", "6 lines and 5 crosses") can move to the general ("one more line than crosses", which we express mathematically as …"crosses +1"). Treating a variable like a number and performing operations on it, such as adding 1, has been identified in the research as a difficult step for students. It can take a long time and much questioning and discussion.

Once the link between the two patterns is established, students who have unlocked more numbers will need to lock them again. This is covered step by step in the **HELP TIM** activity. They will then need to derive the Model Rule.

The aim of this activity is to link patterns. Your task is to build the model by creating the pattern that is missing. You will need to think carefully about how patterns can grow or 'shrink' together, and to think about how to use one pattern to describe the way another pattern changes.

Your task is to turn this: into this:



To get started:

1. Click on the crosses pattern to find the properties box. This has lots of clues that can help you with your pattern. Use the “Place” tab too.

2. Think about how the pattern of blue tiles is connected to the green pattern one. How many blue lines are there for each cross?

Remember to give your model rule (for both the lines and crosses) once you have made your model. Then you’ll get your big green smile!

Theme – Changing the Translation

Activity 1 - Crosses

Title	Crosses
eXpresser Objectives	Understand pattern placement and unlocking numbers
Mathematical Objectives	• translation vectors, • diagonal vs. horizontal/vertical placements, • describing translation/position
Teacher Notes	When a pattern is created, the eXpresser defaults to a horizontal placement with four repetitions. The diagonal positioning of this pattern presents the challenge, as the horizontal/vertical placement needs to be varied. Students will need time to try various combinations, and it is helpful to provide ways to encourage them to describe what is happening when they change the horizontal/vertical components of the vector. For example, note that the default horizontal placement will always place the pattern so that there is just one tile between the end of one building block and the beginning of the next building block. The number in the horizontal placement vector, however, will show the number of squares in the working area needed to translate one tile in a building block to its next appearance in the pattern to achieve that placement. This can be drawn out in discussion initially by asking how far apart the building blocks are placed – answers like “with a gap of 1 tile” or “1 tile apart”, while correct, will need more elicitation as this describes where the building blocks **are**, but that will not allow the eXpresser to “know” where to place each tile in the pattern – to do this, we need to give an instruction about how far apart each tile is repeated. This is part of the movement from observing and describing the model to analysing the structure of the model in a mathematical sense. Answers such as “leave a gap of 5 tiles” need to be rethought, and then rephrased as “repeat every _____ tiles”. A number can be typed directly into the box once the pattern is created, but after the pattern has been created, the number generator needs to be used to change the placement components. (Note that only whole numbers or zero can be used) Set out below are a few thoughts about some other mathematical concepts that this activity could be used for class discussions.

Describing position:

A preliminary discussion of the ways we describe location or placement might be useful to introduce this activity. For example, remind (or explain, depending upon prior knowledge) students that the (x, y) co-ordinates of the Cartesian plane allows us to describe position, and translation vectors are used to describe changes in position and the eXpresser is similar, but different because:

In the eXpresser, the positive direction is downward

Negative placement:
Encourage students to try different numbers: what happens if zero is used in either component of the placement vector? What about a negative number? How would you get a vertical pattern?

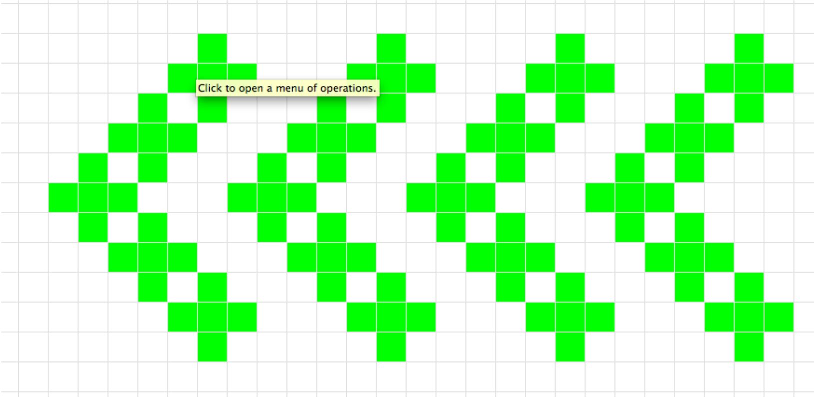
Using negative numbers to describe directional differences is an important aspect of "directed" numbers, and students will come across it with translations. It is, however, often not always made explicit in texts, where the focus may be more on operations on negative numbers.

The Reindeer activity requires students to change the direction of one pattern in a model and is a useful activity to set students after **CROSSES**, perhaps as a homework task.

Zero:
Using negative numbers in this way also highlights the role of zero as the origin, or placeholder, neither one direction nor the other. This could be used to encourage a discussion about how zero does not act like other numbers (for example, we can't divide by it, we get zero when we multiply by it, we get 1 when we raise any number to the power of zero, etc.).

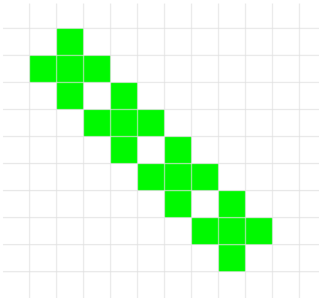
Unlocking:
It may be necessary to remind students that animating in the General Model will show the effect of unlocking a number (that is, the General Model allows them to "keep an eye on the general" because it assigns random values to the unlocked number) while looking at the particular, which is a fundamental design feature of the eXpresser.
When students complete the activity, they can be challenged to create a model that animates in the opposite direction (i.e. upwards and to the left), using negative numbers for the components.

The model below (which shows the 4th term) has been created by copying the finished model (using "Copy" from the menu of operations) and changing some of the placement components. Students can be challenged to create it:



Note that **LINES AND CROSSES** has a model of Crosses embedded, with the task being to create lines in between. Students may be directed to this activity after completing Crosses, and use the Properties of the given Crosses model to check how it has been done, and confirm that their own model is correct.

The aim of this activity is to create a model that animates in a different direction to earlier activities. The task is to create a line of crosses like this:



Think about how to place the pattern after you make a building block (remember to look at the "[Place]" tab) so that the second cross just touches the first, and the third one just touches the second, and so on.

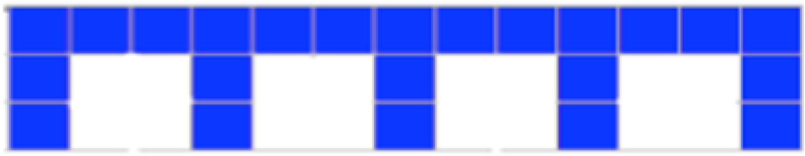
BUILDING MORE COMPLEX MODELS

Activity 1 – Bridges

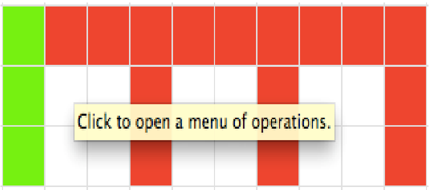
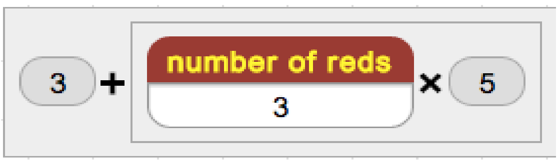
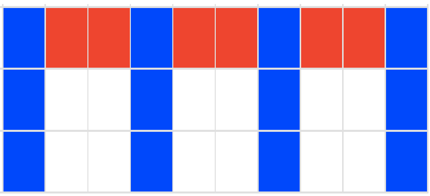
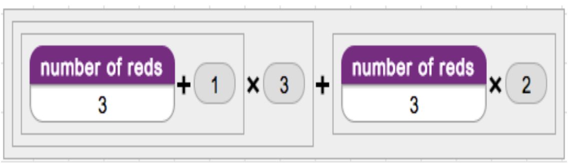
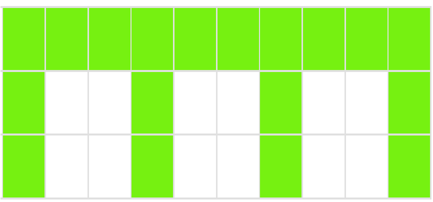
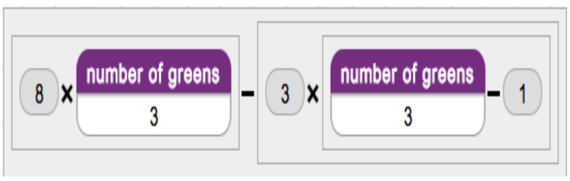
Title	Bridges
eXpresser Objectives	Use patterns to construct models, use more than one colour to show different structures, find a rule for new models
Mathematical Objectives	• Algebraic equivalence
Teacher Notes	Students are asked to construct the Bridges model and use more than one pattern to make the model. Different colours are used for each pattern to show other people how the models are made. It should be possible to find a rule for the number of tiles for any Model Number. • Use pattern(s) to construct the model • Make sure 'My Model' is always coloured • Check that the 'General Mode' animates without messing-up • Make sure the model rule is always correct

<u>Task/Activity</u>

Students are presented with the Bridges Model in blue tiles animating on the left hand side of eXpresser:



Students can come up with a number of different models and their corresponding Model Rules, such as:

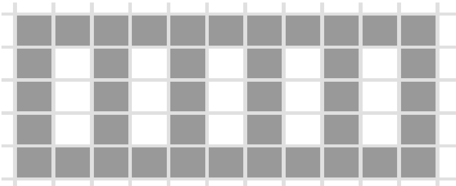
	
	
	

Activity 2 – Traintracks

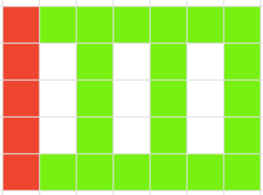
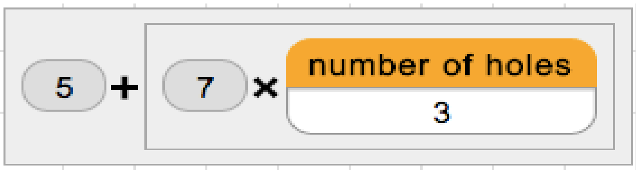
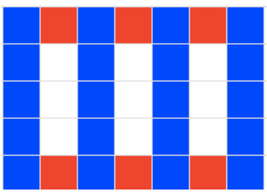
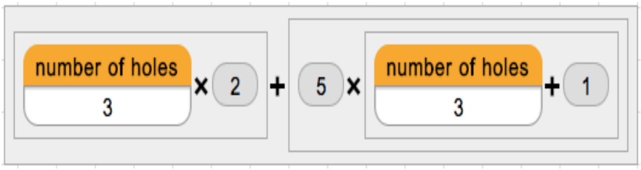
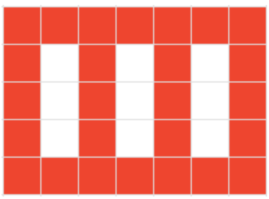
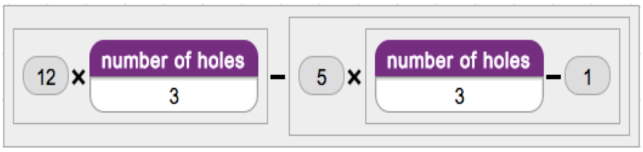
Title	Traintracks
eXpresser Objectives	Use patterns to construct models, use more than one colour to show different structures, find a rule for new models
Mathematical Objectives	• Algebraic equivalence
Teacher Notes	Students are asked to construct the Traintracks model and use more than one pattern to make the model. Different colours for each pattern show other people how to make a model. It should be possible to find a rule for the number of tiles for any Model Number. • Use pattern(s) to construct the model • Make sure ‘My Model’ is always coloured • Check that the ‘General Mode’ animates without messing-up • Make sure the model rule is always correct

<u>Task/Activity</u>

Students are presented with the Traintracks Model in grey tiles animating on the left hand side of eXpresser:



Students can come up with a number of different models and their corresponding Model Rules, such as:

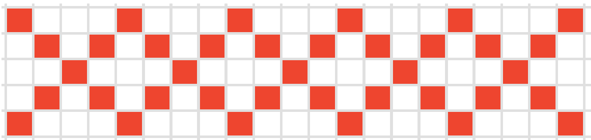
	
	
	

Activity 3 – Stars

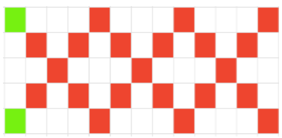
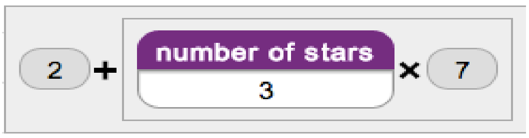
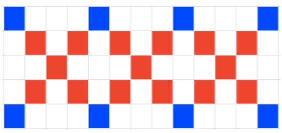
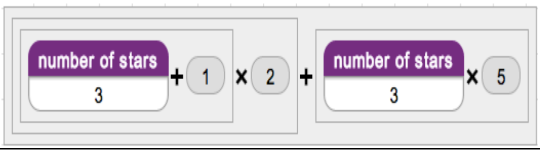
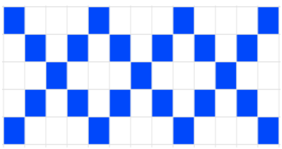
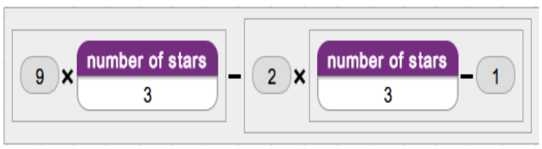
Title	Stars
eXpresser Objectives	Use patterns to construct models, use more than one colour to show different structures, find a rule for new models
Mathematical Objectives	• Algebraic equivalence
Teacher Notes	Students are asked to construct the Stars model and use more than one pattern to make the model. Different colours for each pattern show other people how to make a model. It should be possible to find a rule for the number of tiles for any Model Number. • Use pattern(s) to construct the model • Make sure 'My Model' is always coloured • Check that the 'General Mode' animates without messing-up Make sure the model rule is always correct

<u>Task/Activity</u>

Students launch the Stars Model in red tiles animating on the left hand side of eXpresser and are given the following Goals to achieve:



Students can come up with a number of different models and their corresponding Model Rules, such as:

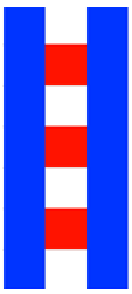
EXTENSION ACTIVITIES

Activity 1 - Ladder

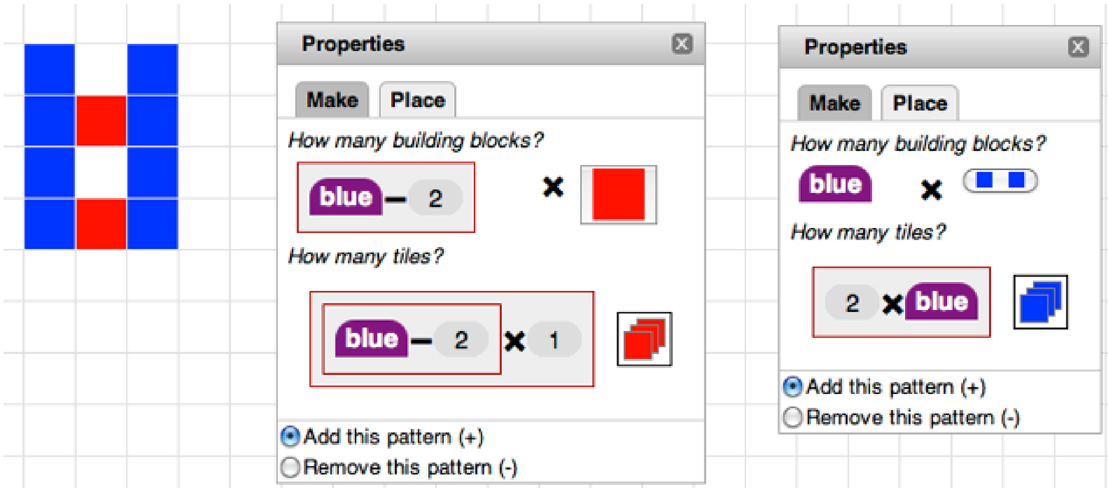
Title	Ladder
eXpresser Objectives	Creating a model with more than one pattern
Mathematical Objectives	• Making variables and sequences
Teacher Notes	This activity is a good follow up activity for Traintracks, either as an extension task or as a homework task. It brings up similar issues of linking building blocks, and there are a number of approaches, so it provides a good source of discussion of equivalence. Focus on the requirement for a rung in each Model number – this will mean that some models will need to have the domain limited (recall that this is done by amending the settings in the slider. Students will inevitably unlock more than one number – encourage them to look carefully at what is repeated as the model animates.

<u>Task/Activity</u>

Students aim to create the Ladder model. Make sure that it works for any number of rungs!



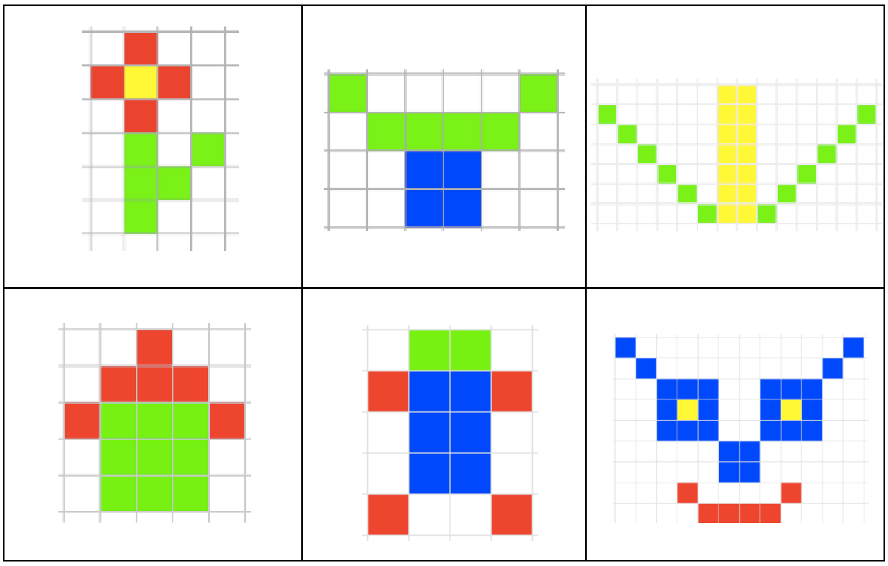
Here is one attempt that did not work.
Can you see what is wrong with it?



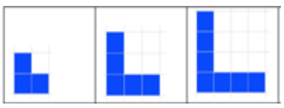
Question: How could this be fixed?

Activity 2 – Make your own building block

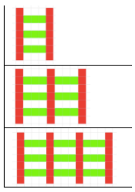
Here are examples of building blocks that can be created:



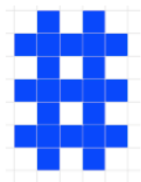
Additional activities aim to take students towards *explicitly* transferring their work on eXpresser to formalising their algebra, by introducing an eXpresser-based context for the development of algebraic skills such as creating and manipulating expressions, substitution and solving equations.



L-SHAPED



FENCES



MODEL MATCH

All of these activities can be worked on individually or in groups. In trials, teachers found small group work on these activities helpful to get articulation of rules and pattern structure, and one-to-one work supported early (or reticent) articulation, such as lower performing students and/or those for whom a transfer from one learning environment to another presents particular challenges.

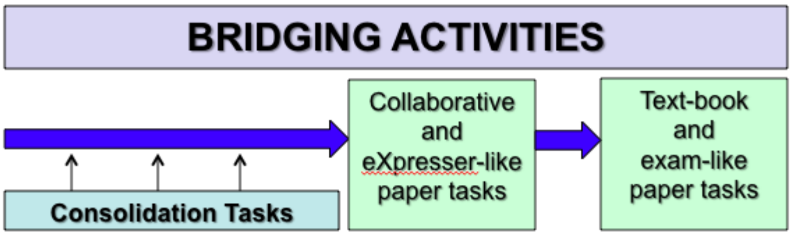
If students are to work individually (for example, as homework or extension exercise), then encourage their work to be prepared for display – as a poster for example. The move from, and between, the structural to the algebraic and numerical takes time, and needs often revisiting. Giving students opportunities to explain their work and to see other students' examples of work and explanations of structure will help consolidate the learning and provide a reference point for future lessons.

BRIDGING ACTIVITIES

These activities are designed to *assess* and *consolidate* learning that has taken place during the activities using eXpresser, and to support the students in making links between the eXpresser work and the algebra they will meet as part of their 'normal' paper-based curriculum.

Some activities are short, open-ended questions and challenges where students are required to apply what they have learnt with eXpresser to pattern-based, or figural sequences. In these questions, the focus is on how students analyse and describe the structures they perceive in figural sequences (rather than algebraic manipulation). Such questions are useful to assist the teacher in assessing the learning that has taken place during the earlier eXpresser activities and in drawing out students' conceptions and constructions of how patterns and their rules might be described.

We have designed four types of bridging activities, which are shown below:



The schematic presentation of the Bridging Activities

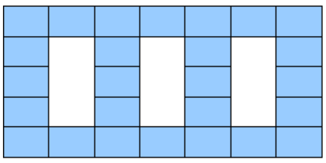
(i) consolidation tasks; short tasks that are used to encourage students to reflect back on their interactions with eXpresser,
(ii) collaborative tasks in which students are asked to decide and then justify if different algebraic rules are equivalent (or not) (see for example one presented on page 50 as an extension task),
(iii) 'eXpresser-like' paper tasks, which are figural pattern generalisation tasks as they had seen on the computer but presented on paper so without the dynamic aspects and links, and
(iv) text-book or examination tasks.

Activity 1 – Traintracks on paper (Consolidation task)

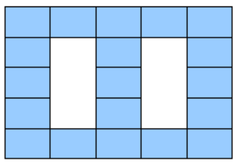
Title	Traintracks on Paper
Mathematical Objectives	• Solve a figural pattern generalisation task and find a general rule • Identify the variables and the constant in the model • Express relations between variables algebraically • Reason mathematically by conjecturing relationships and generalisations • Freely move between different numerical, algebraic and diagrammatic representations • Interpret mathematical relationships algebraically
Teacher Notes	• This activity is a useful consolidation exercise, by way of extension or homework, after doing the Traintracks activity on eXpresser. The aim is for the focus on examining structure when building the model on the eXpresser to be carried over and used when looking at patterns on paper.

<u>Task / Activity</u>

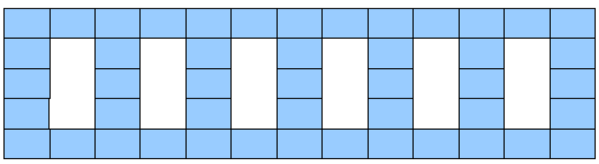
Here are three pictures of the Traintracks on Paper model. Use these, and your work on eXpresser, to answer the questions. In your answers, show how you got to your answer, and show as much working as you can.



Model 3



Model 2



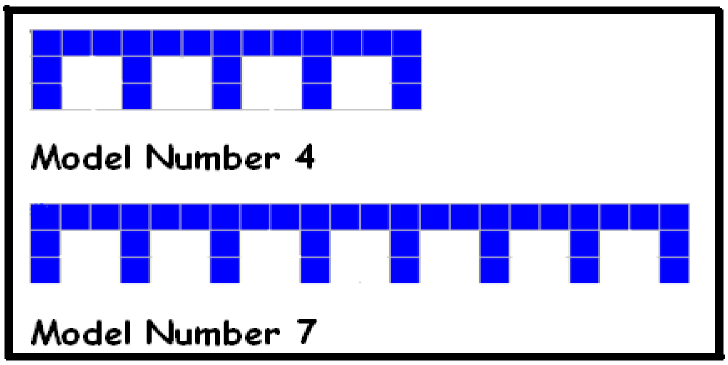
Model 6

1) How many blue tiles would be needed to make Model 12?
2) How many blue tiles would be needed to make Model 50?
3) How many blue tiles would be needed to make Model 1?
4) How many blue tiles would be needed to make Model 200?
5) If we use ‘M’ to stand for the model number, how many blue tiles would be needed to make Model ‘M’?
6) Use the space below to explain the different parts of your rule – use the diagrams provided or draw your own if it helps

Activity 2 – Bridges (eXpresser paper-based task)

Title	Bridges
Mathematical Objectives	• Solve a figural pattern generalisation task and find a general rule • Identify the variables and the constant in the model • Express relations between variables algebraically • Use/Apply a general rule Generate terms of a sequence that are not sequential
Teacher Notes	Variations of this question are common in assessments and offer students the chance to apply what they learned from their interactions with eXpresser on a task, which involves an eXpresser-like model, and teachers the chance to assess students' structural thinking and also identify possible linear scaling issues. For example, students are expected to use their derived rule to answer questions 2, 3 and 4 below. However, our own and our teacher collaborators' experiences has revealed that students often resort to linear scaling and double their answer for Model Number 5 to find the answer for Model Number 10 (Q3) or multiply by 10 the answer for Model Number 10 to find Model Number 100 (Q4). It might also be useful to use this activity to start looking at the numerical sequence that is generated when the total number of tiles is calculated for each model number. This model lends itself to an initial investigation of the numerical representation of the sequence, as the number of blue tiles corresponds to the term, or model number, which makes the arithmetic sequence easier to see.

<u>Task / Activity</u>



1) Find the rule for the number of tiles for any Model Number
2) Find the number of tiles for Model Number 5
3) Find the number of tiles for Model Number 10
4) Find the number of tiles for Model Number 100

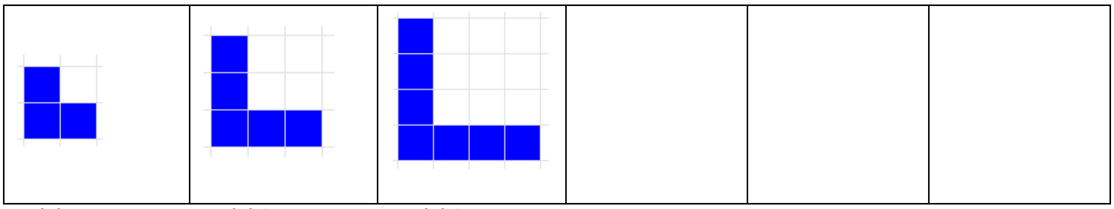
Activity 3 – L-Shapes (eXpresser paper-based task)

Title	L-Shapes
Mathematical Objectives	• Solve a figural pattern generalisation task and find a general rule • Identify the variables and the constant in the model • Express relations between variables algebraically • Simplify and manipulate algebraic expressions to maintain equivalence • Reason mathematically by conjecturing relationships and generalisations and developing an argument using mathematical language
Teacher Notes	This activity consolidates work on eXpresser involving creating expressions and finding rules. It is designed as a classroom activity to be done in groups, so that students can discuss and justify their decisions about how the written rules fit the images and the algebraic expressions, but it can be done with students working individually, in which case provide questioning/ discussion time. Formal algebraic language is introduced, but it is backed up by the construction-based language of eXpresser, such as "growing" building blocks, tiles, model numbers, etc. Prior knowledge of algebraic notation is not necessary, although if students are not familiar with such things as 2L meaning 2xL then the last exercise may best be done in a class plenary. The FENCES activity, which follows, develops the rule-finding theme of this activity, but is more challenging; it is more abstract and differs from the eXpresser concept of using the model number for the variable. Encourage students to try to build the model on eXpresser as a homework activity – it will challenge thinking about the placement of patterns (as the default placement is downwards, whereas the pattern grows upwards), as well as how to link patterns in a model using one unlocked number.

<u>Task / Activity</u>

The aim of this activity is to start describing patterns by using algebra. Patterns can often be **seen** quite clearly in diagrams, and **described in words**. Learning how to describe a pattern **using algebra** is the next step and is an important mathematical skill. It gives us another way to see similarities and differences.

Here are the first three shapes in the L pattern. Draw the next 3 models in the pattern in the empty boxes.



Model 1 Model 2 Model 3

How many tiles high would the 10th model be? ____________________

How many tiles would there be in the 10th model? ____________________

Write a sentence describing how the pattern grows:

____

____

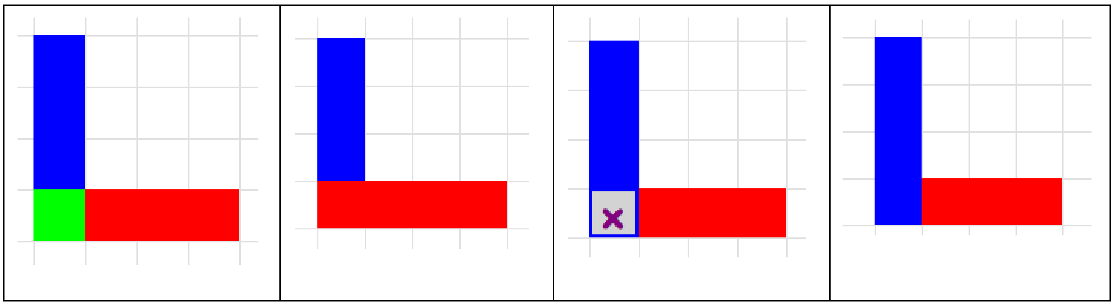
Here are some other students' rules for the pattern. Read the rules and look at the pictures of model number 3 of the pattern below. Write the name of the student below the picture that describes that student's description.

Anna's rule
"There are two patterns, one on the left and one on the right.
The left hand side pattern has one more tile than the model number.
The right hand side pattern has the same number of tiles as the model number."

Bertie's rule
"There are two patterns, one on top of the other.
The top pattern goes up and has the same number of tiles as the model number.
The bottom pattern goes out to the right and has one more tile than the model number."

Cyril's rule
"There is a building block of just one tile and then there are two patterns.
Both patterns have the same number of tiles as the model number. One grows upwards and one grows outwards."

David's rule
"There are two patterns, one growing up and one growing out.
Both patterns have one more tile than the model number.
The patterns overlap on the first tile, so for each model number you have to take away one."



Different configuration for Model Number 3

We can use these rules to find the **total** number of tiles used for different model numbers. For example, using Anna's rule, the 16th model will have 16 tiles in the right hand side pattern and 17 in the left hand side pattern. 17 + 16 = 33.

Use **Bertie's** rule to find the number of tiles in the 19th model:
Use **Cyril's** rule to find the number of tiles in the 12th model:

Use **David's** rule to find the number of tiles in the 25th model:
We can write the rule for the total number of tiles using **algebra**:

If “L” is the model number then, using Bertie’s rule, the number of tiles for any model number can be written as:

$$L \quad + \quad L + 1$$

To find the number of tiles for the 30th model, we **substitute** 30 for L in the rule:

$$30 \quad + \quad 30 + 1 \qquad = \qquad 61$$

Use substitution to find the number of tiles in the 18th model number:

Peter has found the number of tiles in his model number to be 53. What model number did he substitute for “L”?

Look at the rules below. Write the name of the other students below their rule:

L + L + 1	2(L +1) - 1	1 + 2L	L+1 + L
BERTIE			

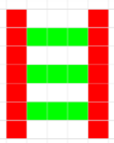
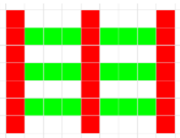
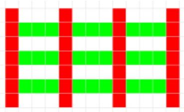
Extension: Rearrange the terms in the rules above to show that each rule can be written the same way.

Activity 4 – Fences

Title	Fences
Mathematical Objectives	• Solve a figural pattern generalisation task and find a general rule • Identify the variables and the constants in the model • Express relations between variables algebraically • Simplify and manipulate algebraic expressions to maintain equivalence • Reason mathematically by conjecturing relationships and generalisations and developing an argument using mathematical language
Teacher Notes	This activity follows on from L-Shaped, and works on developing rules and expressions. The eXpresser metaphor for operating on variables is linking patterns, as nearly all the models in the package are created with just one variable (or “unlocked number” (the exception is the extension activity GRID). The activity HELP TIM guides students through the linking patterns and expressing the repetition of one pattern in terms of another. FENCES focuses on this idea of dependence, and develops expressing one variable in terms of another. Make it clear to students that the “rule” or expression they are finding in this exercise is not a model rule in the eXpresser sense, but a rule connecting the number of posts and the number of rails. (An extension or homework task may be to find the model rule for the total number of tiles in the model.) The focus, however, of this activity is to take the “expression-making” skill that the eXpresser develops in finding model rules beyond finding nth term rules in sequences, and gives a way to connect variables. Substituting different values helps see how the rule works, and is invaluable in getting students to test out their rules. Have students give their rule to others for others, or to the class to test as a group by substituting different values.

<u>Task / Activity</u>

Here are the first three models of the Fence pattern. Look at the number of posts (red) and rails (green). Fill in the table.

Model	Number of posts (*p*)	Number of rails (*r*)
		
		
		

Here are three rules that connect the number of rails and the number of posts in the fences:

Rule A
To find the number of rails, take the number of posts, then subtract 1, and multiply the answer by 3

Rule B
To find the number of rails, take the number of posts, and multiply by 2, then subtract 3 and add the number of posts

Rule C
To find the number of rails, take the number of posts, then multiply by 3 and subtract 3

If we let r stand for the number of rails and p stand for the number of posts, then we can write Rule A in algebra as:

$$r = 3p - 3$$

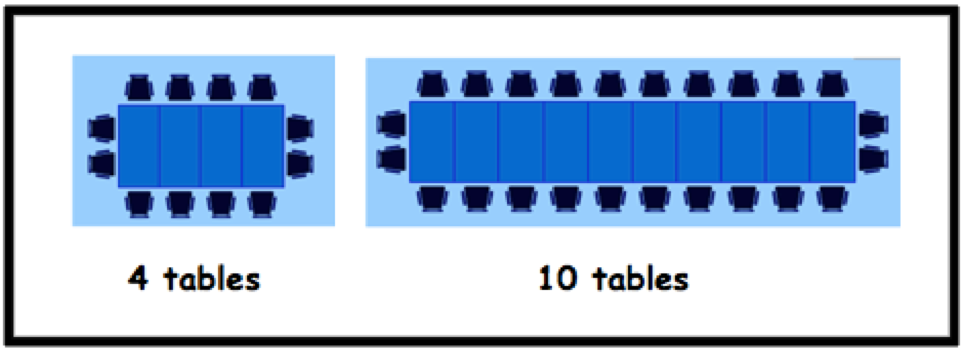
Write the other rules using algebra:

Rule B: $r =$

Rule C: $r =$

Activity 5 – Tables and Chairs

Title	Tables and Chairs
Mathematical Objectives	• Solve a figural pattern generalisation task and find a general rule • Identify the variables and the constant in the model • Express relations between variables algebraically • Use/Apply a general rule
Teacher Notes	Tasks of this type could be given as a homework or final assessment to identify whether students can solve figural pattern generalisation tasks in general.



1) Find the general rule for the number of chairs for any number of tables
2) Use your rule to find the number of chairs for 20 tables
3) Use your rule to find the number of chairs for 200 tables
4) If I have 26 chairs, how many tables do I need?

SAMPLE SCHEME OF WORK

The sample scheme of work presented below, was produced by one of our teacher collaborators and was followed by all teachers in the maths department in their school.

The outline below covers 4 or 5 computer lessons, with plenty of feedback, discussion, written problems and written explanations in between. You may well move faster or slower than this plan. At minimum, students should become familiar with the software and have made successful progress through both 'traintrack' and two other challenging models.

Do make time to discuss the important concepts – I'll note those below where I think they'll arise naturally.

In written tasks I would allow any sensible notation. There is no need to discourage 'number of repetitions x 3' or 'r x 3' in favour of '3r'. They might write 'unlocked number x 3'. Encourage them to think about what the unlocked number is controlling, and whether they'd like to call it something more specific…

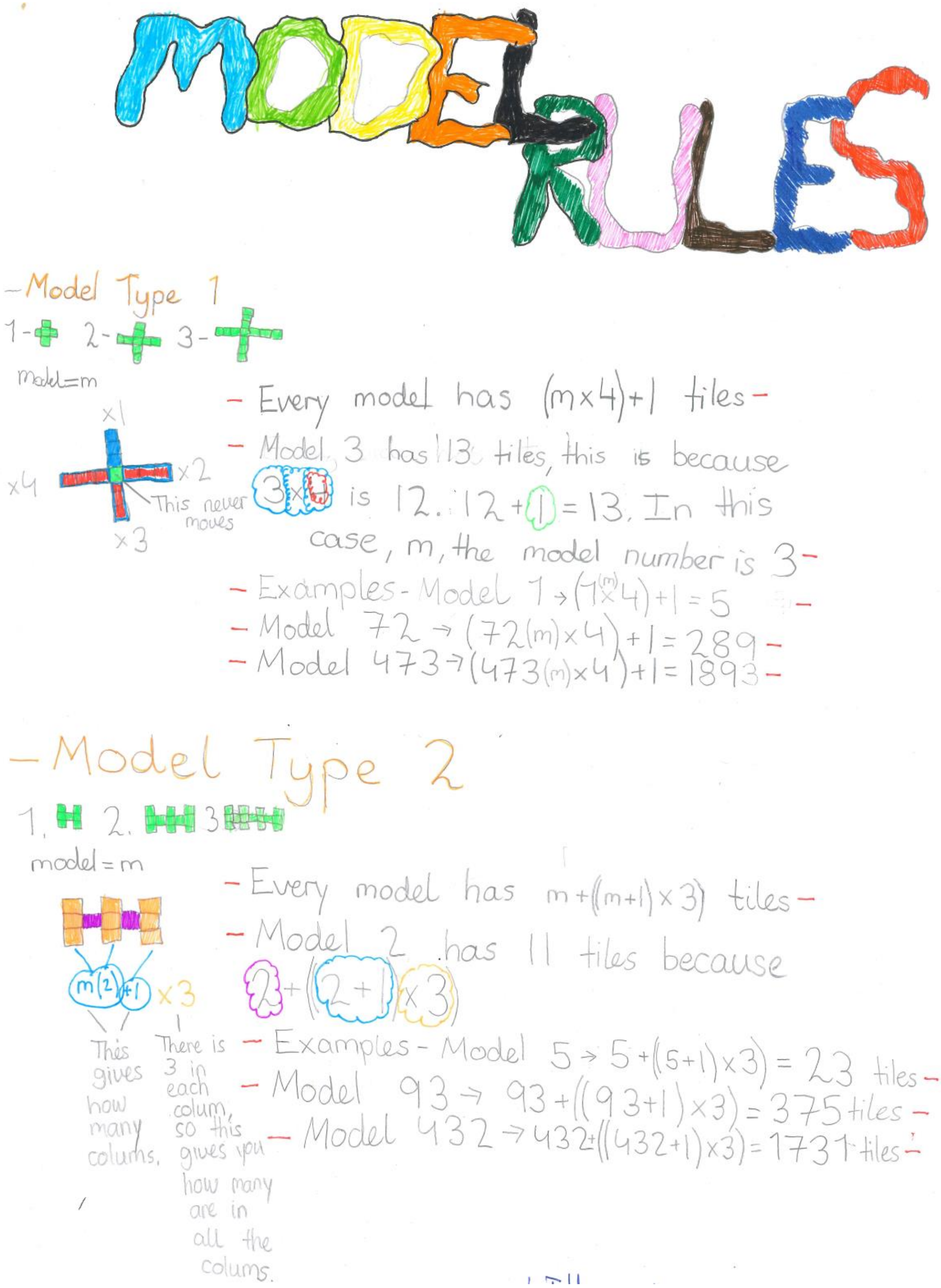
Lesson	MiGen models	Summary	Example lesson plan	Concepts
1	**Free play**	Software fluency: building blocks; patterns; colouring; calculations and 'unclosed' numbers; unlocking numbers; ('negative' tiles).	Lead pupils step-by-step though creating a small image (eg, a smiley face), creating a building block and then a pattern. *(Opportunity for discussion of the translations.)* (NB – to repeat a single tile, you need to make the single tile into a building block first!) Show them how to build calculations by dropping number tiles on top of other number tiles – this can be done on the main page, rather than in the pattern properties box. Demonstrate that '3+5' or '2x4' repetitions works just as well as 8 in the pattern properties box. *(This is a great time to talk about the equals sign as an expression of equivalence, rather than an instruction to write down the answer. Make sure that they're comfortable with statements such as <u>5+2=1+6</u> - maybe get them to fill in some blanks?)*	Translations. Equivalence, unclosed expressions and the equals sign. Generalisation. Dependence.

			Show them how to unlock and (optionally) name a number – can they predict what will happen when they unlock the number of repetitions? The translation vectors? What happens to the colouring in each case? Why? Demonstrate that a copy of an unlocked number can be made by dragging from it. Present the problem of how to keep a pattern coloured when there is an unlocked number. What's the problem? Could you get around that? How might you explain what's changing verbally? How are they working out the correct number of tiles for large numbers? Can they explain the 'rule'? Challenge any unexplained reference to 'nth term' or '3n' – many will have learnt these ideas by rote at previous schools, and have no idea what it actually means. 'What's n?' is usually a question they can't answer beyond saying 'any number', so they will need to engage explicitly with the idea that they are taking the number of repetitions, whatever that happens to be, and doing something to it. They may well start by unlocking the number of tiles separately, rather than copying the number of repetitions – animating and watching the numbers will draw their attention to the fact that these need to always be the same, and then they'll probably need reminding that they know how to do this. Students should try to create an expression using an unlocked number in order to solve this problem – they may need some help/guidance, but encourage as much independence as possible, and certainly a lot of discussion and class feedback. Include some discussion of equivalence of '3 x number' and 'number x 3 'number + number + number', or whatever label they've used.	

			Finish by entering the model rule to colour the general model. If time to fill, you could demonstrate the use of ‘remove this pattern’ to create ‘negative tiles’ which cancel out tiles of that colour. Ask students to create a pattern of 3x3 squares with any translation. Can they create a negative pattern which removes the middle of all of their squares? (These should still be static models.) However, very few students choose to use this feature, and it can be discussed as and when pupils ask about it or start to discuss the need to ‘remove’ tiles.	
2	**Mr Happy’s Hair**	Colouring animated models and linking patterns by repeating use of variable.	Make it clear that they need to create the example model by adding a second pattern to the one given, NOT by altering the given model. Then enter the model rule at the bottom by adding together the two pattern totals. Ext – Can they add a negative pattern to make Mr Happy with hair, but no nose?!	Generalisation. Dependence. Linking variables.
2/3	**Traintracks**	Build traintrack structure, animate, and colour.	Encourage them to use a mixture of repeated and unrepeated designs to create their own structures, then ask them to create harder given structures (structure only, not animated), such as ‘traintrack’. *(They may try to build this tile by tile - let them try! Suggest that they make it one section longer, then another, then another… is there a quicker way for them to do it using the software features? Similarly, if they are counting tile by tile in order to colour the patterns, let them – they’ll hopefully figure out that they can multiply up within a repeated structure, particularly if you suggest a large number of repetitions! Once they’ve started to use a multiplicative method, encourage them to just enter the calculation)* For those that finish quickly, ask whether they can build the pattern in a different way – or if this spans two lessons, ask them to build the model differently the second time around.	Algebraic equivalence, expressions, variables and constants, generalisation.

			At end of lesson, leave time to discuss different models and different rules. Perhaps get pairs to present their methods and final model rules – compare these and encourage discussion of the equivalence of different rules. Talk explicitly about the parts of the rule which change and those which don't. Suggest structural changes to the pattern and ask students to suggest how their rule would need to change. HW or end of lesson – written Traintrack task which consolidates structural approach to generalisation and encourages written explanation/justification (see Traintrack task on Y-Drive).	
4-7	**Lines and Crosses** **Stars** Extension task: Try some non-linear patterns	(at least two with computers, up to two looking at written problems)	Attempt increasingly more complex models – for those who are confident, encourage them to build Lines and Crosses (or similar) using two repeated patterns and no static parts – this will require them to have one more repetition of lines than of crosses. By this point, it's important to move regularly between computer-based problems and written tasks – they needn't necessarily bear any relation to each other (although it's helpful if the first couple do), but they should be encouraged to imagine trying to build models in order to figure out the repeated and static elements, and then to use annotated pictures to explain their general rules. Consider some simple non-linear problems too. Even if they can't build 'growing house', can they use the structure of the model to generalise the number of tiles of each colour/in total? Given a rule (eg, 5n+2), can they create building blocks which adhere to it?	

EXAMPLES OF STUDENTS' WORK



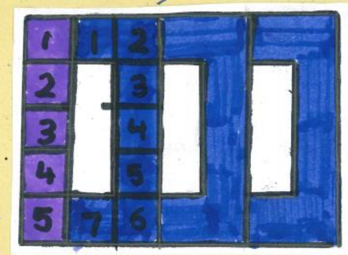
TRAIN TRACK...

Calculating the formula of the pattern:

1. To calculate the formula of the train track, you must start by splitting the pattern into 2 sections: (right) additional building block and the repetitive pattern building block. In the example given the repetitive building block is the C shaped pattern which is repeated constantly at random (the rest of the pattern is not shown).

2. In this example we have decided to give the C shaped building block a name: Blues. This patter contains 7 tiels. Therefore, if the pattern should consist of (model number) m number of tiels, the formula would be: $7 \times M = 7M$ (if $M=3$, then $7M=21$).

repetitive pattern building block.



additional building block.

Model

3. Given that the model isn't only made by Blues, the starting/ending 5 purple tiles must be added to the starting formula. Therefore, the formula for finding the overall number of tiles in the model is: $7M+5$.

How and where to create patterns like these:

You can create many more patterns like thease just by going online on a website called Expresser. There can be many more ways to make many different patterns just by using some colourful squared tiels.

block PATTERN

Model 1:

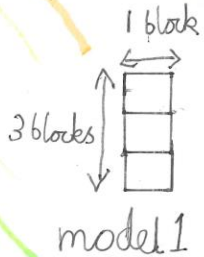
total:3

Model 2:

total:8

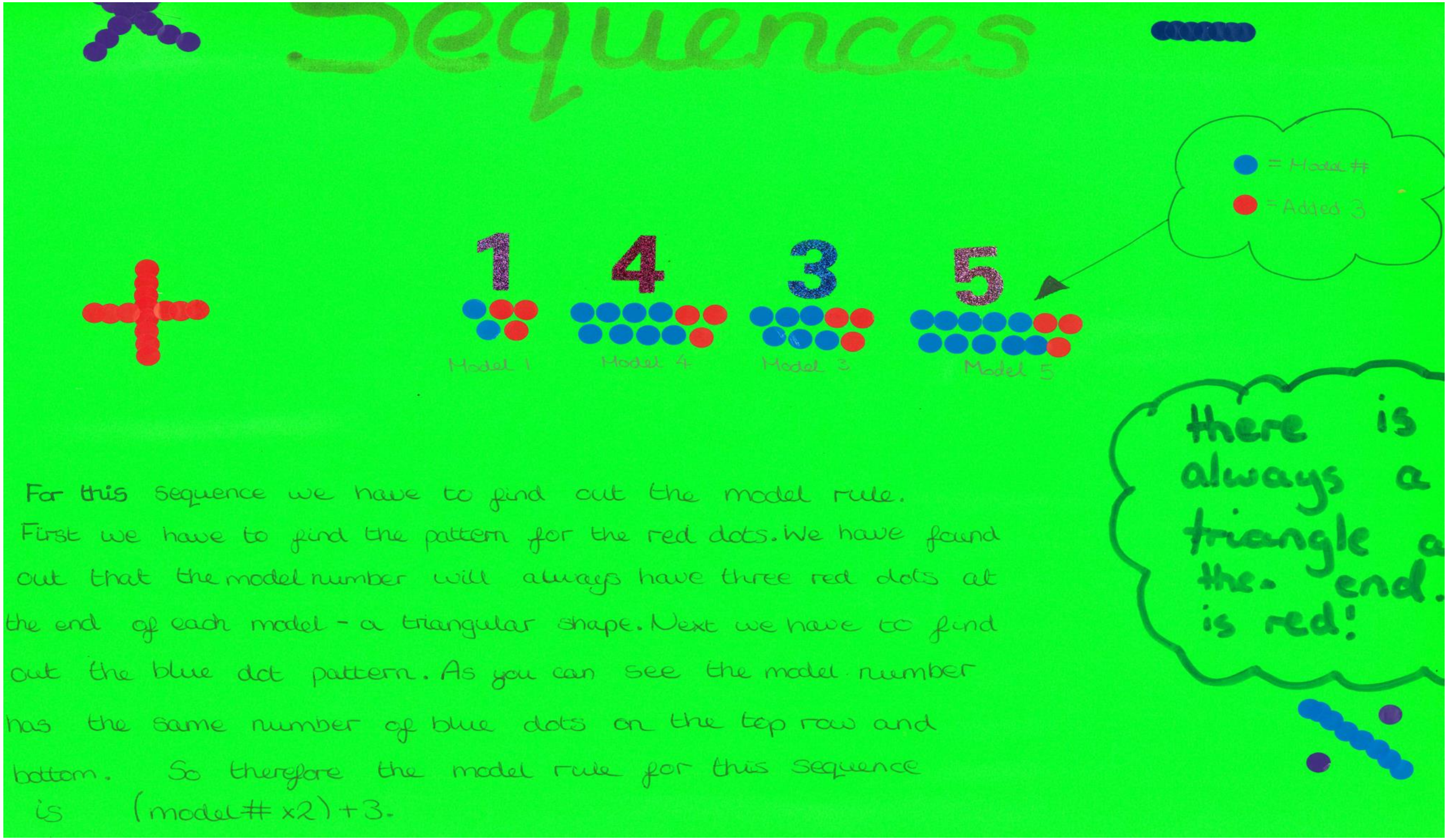
Model 3:

total:15



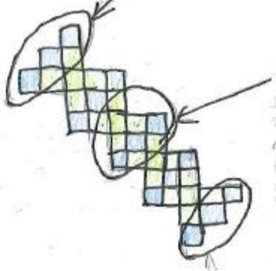
explanation:

The width of any figure in the pattern is alway equivalent to its model number (in this case 1). The height of any figure in the partern is always the model number+2. If you multiply the width by the height of any figure in the pattern, you get the number tiles in the figure. Therefore, the formula is $m \times (m+2)$.


Sequences
1
4
3
5
= Model #
= Added 3
Model 1
Model 4
Model 3
Model 5
there is always a triangle at the end. is red!
For this sequence we have to find out the model rule.
First we have to find the pattern for the red dots. We have found
out that the model number will always have three red dots at
the end of each model - a triangular shape. Next we have to find
out the blue dot pattern. As you can see the model number
has the same number of blue dots on the top row and
bottom. So therefore the model rule for this sequence
is (model # x2) + 3.

Lines + Crosses

This problem involes green crosses and blue lines; the green crosses consist of 5 green tiles and blue lines are made of 3 blue tiles However, you only begin with the crosses as a pattern, and you have to work out how to put the blue lines in the correct position without altering the amount of sliders/patterns.



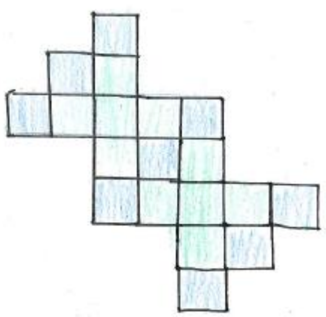
How to do it?

First, make a line of three building block tiles. Now leave them. Make another exactly the same, now click on show properties, make them 2 across and 2 down. Now replace the 4 under 'How many building blocks' for No. of crosses. Then make 'How many tiles' 3 x No of crosses. Now try and increase and decrease the slider and the lines and crosses move together.

Model Rules

For the Model Rules you must do No. of crosses x 8 + 3 because there are eight tiles in a cross + a line; 5 in a cross and 3 in a line, and 3 for the extra line at the top of the pattern.

$M \times 8 + 3$ = Formula



(NO. OF CROSSES X 8)

+ 3

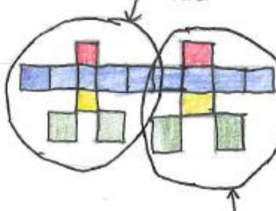
Multicoloured Humans

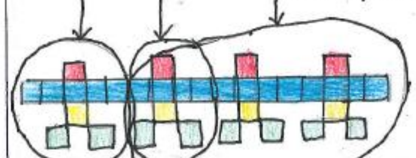
This problem involves every coloured block in expresser – which are blue, red, yellow and green – to make a human. It's not too easy and not too hard. Furthermore, it is made using 1 red, 5 or 4 blue, 1 yellow and 2 green blocks. Similar to the lines and crosses problem, you can only have 1 slider/pattern.

How to do it?

First of all, you need to make a human as shown in picture one then make this a building block. Now repeat what you did but don't put the furthest to the left blue tile in. Make that a pattern and make sure the left blue block touches the blue block next to it. Unlock the number of patterns, call it something, put that in the red, blue, green and yellow and multiply them by how many tiles there are in one pattern. When you increase the pattern, you will see it works.

Picture 1.





Model Rules

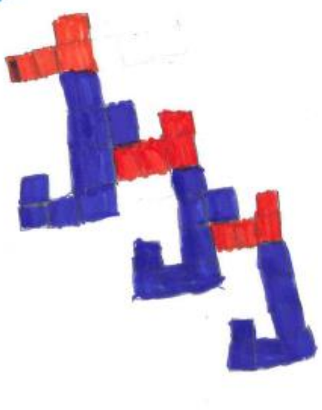
We will be using M to represent the amount of patterns in Multicoloured humans. To find out how many blue blocks there are you would do $(M \times 4)+5$ because there are 4 in one pattern and the extra 5 at the beginning that will always be there. It is the same with the others, red would be $(M \times 1)+1$ or $M+1$, yellow would be $(M \times 1)+1$ or $M+1$ and green would be $(M \times 2)+2$. To work out how many blocks there are in total, you should do $((M \times 4)+5)+((M \times 1)+1)+((M \times 1)+1)+((M \times 2)+2)$ or $(4m+5)+((m+1)\times 2)+(2m+2)$

LINES AND CROSSES

In lines and Crosses there was a pattern which went a diagonal line of blue and then a green cross ect. The blue line always had to be at the begining and at the end of each cross. The bottom line of the cross would have to be the top line of another cross. To make this happen we had to make one separat ~~blue~~ building block at the top and then have a green cross with a blue line at the bottom as another building block. This way there would be a blue line

The formula (with pattern) for the lines and crosses pattern is ~~[illegible]~~ $8x-3$, x being the number of the pattern. The formula for the blue blocks is $3x+3$. The formula for the green blocks is $5x$. The way you made the pattern on expresser was to make a separate building block for the green and blue blocks. Then you ~~went into~~ ~~go into~~ ~~[illegible]~~ unlock the pattern. To make ~~in~~ the blocks coloured go to properties and types in how many blocks you want coloured.

THE SEAHORSE PATTERN



The Seahorse pattern formula is $12x$. The formula for the red blocks is $4x$. The formula for the purple blocks is $8x$. The pattern is basically made out of red and purple blocks. one Seahorse pattern is made out of ~~red~~ ~~[illegible]~~ 3 red blocks and 8 purple blocks. In a ratio this is 3 red : 8 purple.